Título: QUALITATIVE PROPERTIES OF STATIONARY STATES OF SOME NONLOCAL INTERACTION EQUATIONS

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Resumen: In this dissertation, we study the stability of stationary states for some interaction equations and for fragmentation and swarming models. All these models share the common property of nonlocality and the existence of a Lyapunov functional. In the case of the interaction equations and the models for swarming that we consider, they have in common a nonlocal interaction term given by the convolution of the interaction potential, and the density of particles in space. The case of the fragmentation equations is a bit different: they are integro-differential equations, with the nonlocal term given by the fragmentation operator, an integral of a kernel against the density of particles.

We start with an introduction to aggregation equations, with repulsive-attractive radial interaction potential. We derive some existence results and convergence to spherical shell stationary states. One can look for local minimizers of the interaction Lyapunov functional in order to find stable stationary states of the equation. We study radial ins/stability of these particular stationary states. For these aggregation models we will make use of

the gradient flow structure that they have. Confinement properties of solutions of aggregation equations under certain conditions on the interaction potential are studied in Chapter 3. We show that solutions remain compactly supported in a large fixed ball for all times. We continue our research in aggregation equations in Chapter 4, where we characterize the dimensionality of local minimizers of the interaction energy.

Another problem that we study is the asymptotic behavior of growth-fragmentation models. In Chapter 5, we give estimates on asymptotic profiles and a spectral gap inequality for growth-fragmentation equations. These models are not a gradient flow of a particular energy functional. However, they have a Lyapunov functional that we use to prove exponentially fast
convergence of solutions to the asymptotic profiles by showing an entropy - entropy dissipation inequality. This technique gives us stability of the stationary states proving convergence to the local minimizers and it allows for estimates on the rate of convergence to equilibrium.

We finish this thesis with the results in Chapter 6, where we study two second order particle systems for swarming. We refer to these systems as individual based models (IBMs), which is the common language used in swarming. We prove the stability of two particular solutions: flock rings and mill rings. We relate the stability of these ring solutions of the second order models with the stability of the rings of a first order model, the discrete version of the aggregation equation of Chapter 2.