TESIS DOCTORALES - TESEO

Título: EXISTENCE AND MULTIPLICITY OF SOLUTIONS OF FUNCTIONAL DIFFERENTIAL EQUATIONS

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Fecha de lectura: 22/06/2015

Mención a doctor europeo: concedido

Programa de doctorado: D1071V01 Programa de Doctorado en Matemáticas

Dirección:

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Descriptores:

> ECUACIONES DIFERENCIALES ORDINARIAS

> PROBLEMAS DE CONTORNO

> RESOLUCION DE ECUACIONES DIFERENCIALES ORDINARIAS

> ALGEBRA DE OPERADORES

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> 2015fernaexist.pdf

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Resumen: The first part of the memory goes through those discoveries related to Green's functions. In order to do that, first we recall some general results concerning involutions which will help us understand their remarkable analytic and algebraic properties. Chapter 1 will deal about this subject while Chapter 2 will give a brief overview on differential equations with involutions to set the reader in the appropriate research framework.

In Chapter 3 we start working on the theory of Green's functions for functional differential equations with involutions in the most simple cases: order one problems with constant coefficients and reflection. Here we solve the problem with different boundary conditions, studying the specific characteristics which appear when considering periodic, anti-periodic, initial or arbitrary linear boundary conditions. We also apply some very well known techniques (lower and upper solutions method or Krasnosel'skii's Fixed Point Theorem, for instance) in

order to further derive results.

Computing explicitly the Green's function for a problem with nonconstant coefficients is not simple, not even in the case of ordinary differential equations. We face these obstacles in Chapter 4, where we reduce a new, more general problem containing nonconstant coefficients and arbitrary differentiable involutions, to the one studied in Chapter 3.

To end this part of the work, we have Chapter 5, in which we deepen in the algebraic nature of reflections and extrapolate these properties to other algebras. In this way, we do not only generalize the results of Chapter 3 to the case of n-th order problems and general two-point boundary conditions, but also solve functional differential problems in which the Hilbert transform or other adequate operators are involved.

The last chapters of this part are about applying the results we have proved so far to some related problems. First, in Chapter 6, setting again the spotlight on some interesting relation between an equation with reflection and an equation with phi-Laplacian, we obtain some results concerning the periodicity of solutions of that first problem with reflection. Chapter 7 moves to a more practical setting. It is of the greatest interest to have adequate computer programs in order to derive the Green's functions obtained in Chapter 5 for, in general, the computations involved are very convoluted. Being so, we present in this chapter such an algorithm, implemented in Mathematica. The reader can find in the appendix the exact code of the program.

In the second part of the Thesis we use the fixed point index to solve four different kinds of problems increasing in complexity: a problem with reflection, a problem with deviated arguments (applied to a thermostat model), a problem with nonlinear Neumann boundary conditions and a problem with functional nonlinearities in both the equation and the boundary conditions.

As we will see, the particularities of each problem make it impossible to take a common approach to all of the problems studied. Still, there will be important similarities in the different cases which will lead to comparable results.