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Three Essays on

Executive Stock Options

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"No era el hombre más honesto ni el más piadoso,
pero era un hombre valiente. . . "

Arturo Pérez Reverte

(El Capitán Alatriste)

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Introduction

This thesis goes deeply into the study of the executive stock options (ESOs henceforth). The interest to analyze the ESOs is twofold. First, the use of ESOs as an instrument of variable compensation has grown significantly during the last decades, and nowadays, they are the largest component in the CEO's compensation packages. Therefore, to understand the effects and incentives of this instrument becomes relevant. Second, new accounting requirements force firms to recognize ESO issues as a cost using the fair value. However, traditional option pricing formulae like Black-Scholes are not suitable because ESOs present some different features which complicates their valuation. Then, a new stream in the option pricing literature has attempted to propose methods to calculate the fair value of the ESOs according to the accounting standards. Thus, to gain an insight about the fair value is a question of interest.

This thesis can be separated in two parts. The first part comprises the next chapter, and deals with the fair value of the ESO for accounting purposes. The second part contains the last two chapters, and they handle the ESO valuation from the executive point of view considering his endowments, restrictions and risk preferences.

In Chapter 2, we implement a flexible simulation-based approach for the fair value of ESOs that accounts for the vesting period, departure risk and voluntary suboptimal early exercise. We introduce GARCH effects on the underlying asset and we analyze the price bias with respect to the constant volatility case. We also perform a sensitivity analysis with respect to changes in several ESO characteristics. Moreover, we compare this valuation with FAS 123 method revealing a FAS overvaluation. Finally, we value a real ESO plan providing the confidence intervals for the estimated ESO prices due to the need to estimate previously the volatility models.

In Chapter 3 we present an algorithm that combines simulations with the certainty-equivalence principle to obtain the value of an ESO from the perspective of an undiversified and risk-averse executive. We verify numerically the algorithm comparing our results with the ones from previous articles. We show how the early exercise decision depends on the executive risk-aversion and diversification degree. Our algorithm, compared with the previous models, has the advantage that is based on simulations, then we relax the common assumption of constant parameters, introducing time varying volatility in a markov regime switching framework, obtaining significant biases respect to the constant volatility one.

Chapter 4 extends the algorithm presented in Chapter 3 to consider the two state-variable framework (firm stock price and market portfolio price). We conduct a sensitivity analysis revealing the effects of the dividend yield, market risk-remium, market correlation and vesting period in the subjective ESO valuation, the expected exercise time and in the executive's portfolio composition. We obtain the fair value of the ESO under this framework and we account for the difference between executive's valuation and company cost of the ESO. Finally, we compute the subjective incentives depending on the ESO moneyness.

2

American GARCH Employee Stock Option Valuation

2.1 Introduction

Recent developments of accounting standards require firms to account for employee stock option (ESO) grants using a fair value method and to recognize this value as a cost. The International Accounting Standard Board (IASB) publishes the International Financial Reporting Standard 2, *Share-Based Payments* (IFRS 2). This standard has been adopted in the European Union since 2005 and it is obligatory for all traded firms. The IFRS 2 is the first standard that forces firms to recognize all share-based payments, including ESO, as an expense and a fair-value based method is required in the standard. However, how to calculate the ESO fair value has been very discussed and it has become one of the major difficulties in accounting because IFRS 2 does not specify which pricing models should be used.

The IFRS 2 only describes the factors that should be taken into account when estimating the fair value (IFRS 2 paragraphs B4 to B10). Specifically, the IFRS requires a valuation model which considers the exercise price of the option, the time to maturity, the current stock value, the expected volatility, dividend yield and risk-free rate for the option life, the vesting period and the possibility of an early exercise. This standard also suggests that formulae like Black-Scholes (1973) may be not suitable because in the Black-Scholes paradigm volatility and other parameters are not allowed to be time varying (IFRS 2, paragraph B5). In this work, we assume that possible dilution effects are incorporated in the stock price on the grant date, which is in line with the IFRS 2 (paragraph B40).

The Financial Accounting Standard Board (FASB) revised the financial accounting standard 123 (FAS 123R) in order to be compatible with the IFRS 2. Like IFRS 2, the FAS 123R does not specify a preference for a particular valuation technique or model in estimating the fair value, although it enumerates the factors required in the valuation technique at a minimum (FAS 123R, paragraph A18). These factors are rather the same as those the IFRS 2. The FAS 123R valuation, like IFRS 2, is applicable to the ESO value from the point of view of the issuing company (non-constrained agent), which is the aim of this paper.

To be more precise, we compute the ”ESO objective value” which is the cost to the firm of issuing the option and lies between the ”market” and ”subjective” values. The “market value” is the option value if the option is held by an unconstrained agent. The “subjective value” is the option value for the holder under a situation of undiversified portfolio and suboptimal option exercise from the market's perspective[1]. Finally, the “objective value” is computed by recognizing the suboptimal exercise but not the discount due to the lack of diversification. Thus, this price is identified as the fair value of ESO grants[2]. This is the amount the firm would have to pay to an unconstrained outside institution or investor to assume their short position.

Moreover, the valuation technique should be based on established principles of financial economics like time value of money and risk-neutral valuation (FAS 123R, paragraph A8). In a footnote, the FAS 123R recognizes that Monte Carlo simulation technique is a valuation method that may satisfy the requirements of the standard. A method of estimating volatility is not specified in the standard, but it provides a list of factors that should be considered in the estimation proce-

[1]The subjective valuation of ESO is analyzed in the remaining chapters.
[2]For more details see Ingersoll (2006).

dure. Among others, it should include volatility changes with mean reversions (FAS 123R, paragraph A32).

If we compare ESOs with conventional traded options, it is shown that ESOs exhibit several different features[3]. They usually have an initial vesting period during which the exercise is not allowed. The ESO holder is also subject to a departure risk. If he leaves the firm, either voluntarily or not, he must exercise immediately the ESO though it may be suboptimal. Nevertheless, if the departure occurs during the vesting period, the employee loses the ESO. In contrast with conventional options, ESOs are not transferable. Furthermore, the holder is not allowed to hedge his ESOs by taking short positions in the company's stock. Section 16-C of the Securities Exchange Act prohibits insiders from selling their firm's stock short. Because of either diversification or liquidity reasons, the employee could exercise the ESO suboptimally in contrast to being a tradable American-style option[4]. The related empirical evidence of this fact can be found, among others, in Huddart and Lang (1996), Carpenter (1998) and Bettis et al. (2005). In our work, we suppose that the distribution of the future stock price is independent of the ESO holders' decisions[5]. In short, as accounting standards establish, the fair-value based method should incorporate, at least, the stylized facts of vesting period, departure risk and suboptimal early exercise.

Many models have been developed in order to estimate the fair value of the

[3]For a detailed discussion about the differences between standard options and ESO grants, see Rubinstein (1995).

[4]Other possible reasons for this suboptimal exercise are taxes or inside information, see Carpenter (1998).

[5]This assumption is also considered, among others, by Hall and Murphy (2002), Tian (2004) and Cai and Vijh (2004). A possible justification for this assumption is as follows. Most companies grant options under some established compensation policy, so the incentive effects of options may be largely incorporated into stock prices as of the grant date.

ESO plans. Jennergren and Näslund (1993) use the Black-Scholes (1973) framework to get the extended partial differential equation (PDE) for an option that includes the early exercise as an exogenous stopping time measured by the first jump time of a Poisson process with constant intensity. Carr and Linetsky (2000) develop an analytical specification based on the stochastic intensity framework in which the intensity or exit rate can be decomposed into two parts: the involuntary and voluntary exercises measured by a constant and a function of the stock price path respectively. Pandher (2003) extends the intensity-based model incorporating multiple severance risks and modeling the severance event as a doubly stochastic poisson process.

Huddart (1994) and Kulatilaka and Marcus (1994) develop a binomial tree model to value the ESO firm cost using a utility-based framework for determining the agent's exercise policy. Nevertheless, the authors recognize that their model cannot be used in practice since it requires some input variables that are difficult to estimate, such as employee risk aversion, non-option wealth of the employee and so forth. Tian (2004) also uses a utility-based framework and the certainty-equivalence of the ESO to analyze the incentives of the ESO plans. He only considers the European-style ESO case without departure risk. All these models are analyzed more deeply in Chapters 3 and 4.

More recently, Hull and White (2004) propose a binomial approach to calculate the ESO's fair value where the early exercise behavior is modeled as a barrier. A similar approach is presented by Ammann and Seiz (2004) such that the early exercise behavior is here modeled adjusting the strike price of the option. The continuous version with barrier can be found in Ingersoll (2006).

Sircar and Xiong (2007) provide a not fully analytical valuation for a perpetual American-style ESO taking into account the resetting and reloading provisions that are features of many option programs. Finally, Cvitanic et al. (2008) obtain a closed-form expression for pricing ESOs such that the rate of exit is modeled following the same approach in the default bonds literature, and the early exercise effect is captured through a barrier.

Our main contribution is valuing typical long-dated American-style ESOs when the volatility of the underlying asset is assumed to be time-varying under the GARCH framework. This analysis becomes interesting in obtaining the ESO price difference when constant volatility (incorrect model) is assumed erroneously. Note that the ESO costs reported in the statements depend on the manager's volatilty model choice which may not be the one with the best performance[6]. We use the methodology of Duan (1995) to get the locally risk-neutral measure for the ESO valuation. We will concentrate here on pricing American GARCH options and consider the main ESO features mentioned before.

Our work consists of an extension of Stentoft (2005), that is based on the least-squares simulation approach of Longstaff and Schwartz (2001), for the case of ESO valuation. In short, our proposed valuation method based on simulations is in line with IFRS 2 and FAS 123R because it allows for a time varying volatility, besides the vesting period, the departure risk and the early exercise behavior. Duan and Wei (2005) also use the GARCH framework to value ESOs. However, they are more interested on the executive's incentive effects due to the underlying asset risk composition (systematic vs. non-systematic) for a European-style

[6]Yermack (1998) shows that managers systematically reduce (manipulate) firm's historical volatility levels.

ESO without departure/cancellation risk.

For the last decades, empirical evidence supports that conditional volatilities of many daily stock-index returns exhibit long-memory[7], that is, shocks to the conditional volatility die away at a hyperbolic rate instead of the exponential rate (short-memory) under the popular GARCH structure. As expected, this feature affects the option valuation leading to significant price biases when modeling the conditional volatility of the underlying daily return under a short-memory structure instead of a long-memory one. Because of it, we consider alternative volatility dynamics for ESO pricing to capture the mispricing effects with respect to a naive model (constant volatility).

The rest of the chapter is organized as follows. Our valuation method is explained in Section 2.2. Section 2.3 shows some numerical results based on a simulation analysis. The implications for the accounting standards are explained in Section 2.6. Section 2.7 is about pricing a real case of an ESO plan providing the corresponding price confidence intervals due to the error introduced by the need to estimate GARCH parameters. Finally, Section 2.8 concludes.

2.2 ESO valuation with GARCH type volatility

In this section, we first introduce some specifications for the volatility under the GARCH context for the ESO pricing and second, we implement the algorithm to obtain this fair value based on an American option under several restrictions.

[7]See, for instance, Baillie et al. (1996) and Hyung et al. (2006).

2.2.1 GARCH framework

The famous Black-Scholes (1973) formula for valuing European options assumes that the underlying stock price S_t follows a lognormal distribution and hence, the continuously compounded daily logarithm return, $R_t \equiv \ln(S_t/S_{t-1})$, is Normal distributed. Nevertheless, empirical evidence suggests that the unconditional distribution of daily returns exhibit some stylized facts like fat tails and asymmetries which lead to a clear rejection of the normality assumption of stock returns. Moreover, it also holds the clustering phenomenon for the estimated volatility series, that is, a high (low) volatility period is followed by a high (low) volatility one. This pattern is well captured by the ARCH model of Engle (1982), and the GARCH model proposed by Bollerslev (1986) and Taylor (1986). Under the GARCH framework, the conditional variance is a function of past conditional variances and past innovations. The simple GARCH model for R_t depending only on the past squared innovation and the past conditional variance is formulated under the real-world measure, or $\mathbb{P}$–meausure, as

$$R_t = r - d + \lambda\sigma_t - \frac{1}{2}\sigma_t^2 + \xi_t; \qquad \xi_t = \sigma_t z_t \tag{2.1}$$

$$\sigma_t^2 = \omega + \alpha\xi_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{2.2}$$

where z_t is assumed to be i.i.d. $N(0,1)$ and hence, $\xi_t|\mathcal{F}_{t-1} \sim N(0,\sigma_t^2)$ such that $\mathcal{F}_{t-1}$ is the time $t-1$ information set defined as $\mathcal{F}_{t-1} \equiv \{\xi_{t-i}, i \geq 1\}$. The parameter r is the daily constant continuously compounded risk-free rate. The parameter d is the daily continuously compounded dividend yield[8] and λ is the

[8] Both r and d are expressed in terms of the yearly parameters r_f and D respectively, i.e. $r = r_f\delta t$ and $d = D\delta t$, where δt represents one day.

market price of risk. Equation (2.2) is the so-called GARCH(1,1) specification with the three parameters restricted to be positive and a persistence level of $\alpha + \beta < 1$ for covariance stationarity.

Under the $\mathbb{P}$–measure, the unconditional variance for the daily return in equation (2.1) given (2.2) is obtained as $\mathbb{E}^{\mathbb{P}}\left[\sigma_t^2\right] = \omega/(1-\alpha-\beta)$. The conditional expectation of S_t from the return equation (2.1) is

$$\mathbb{E}^{\mathbb{P}}\left[S_t|\mathcal{F}_{t-1}\right] = S_{t-1}\exp(r - d + \lambda\sigma_t) \tag{2.3}$$

and the conditional variance of R_t becomes

$$Var^{\mathbb{P}}\left[R_t|\mathcal{F}_{t-1}\right] = \sigma_t^2 \tag{2.4}$$

The conditional distribution of the one-period return is Normal distributed but the unconditional one exhibits a kurtosis higher than three, that is, it allows for the fat-tailed phenomenon documented by some empirical evidence. This model becomes a good candidate to capture the time-varying volatility of financial series under a very parsimonious way.

Taylor (1986) and Ding et al. (1993) show that the absolute value of returns has the long-memory property, that is, the sample autocorrelation function (ACF) of either $|R_t|$ or R_t^2 —as a proxy of the volatility or variance respectively— remains significantly different from zero even at long lags. Note that the GARCH model implies an exponential (fast) decay rate instead of a hyperbolic (slow) decay rate agreed with the long-memory feature for the ACF of ξ_t^2 in equation (2.1). As expected, this feature affects the option valuation leading to significant price biases when modeling the conditional volatility of the underlying daily return under a short-memory volatility structure instead of a long-memory one[9]. We also get the ESO valuation under the long-memory volatility specification C-GARCH of Engle and Lee (1999), that becomes a restricted GARCH(2,2) process. We select this model against the popular Fractional GARCH models, introduced among others by Bollerslev and Mikkelsen (1996, 1999) and Taylor (2000) for option valuation, since the former is easier to implement while the last are somewhat cumbersome.

The C-GARCH structure of Engle and Lee (1999) can be decomposed into a permanent (long-run) component, denoted as q_t in equation (2.6), and a transitory (short-run) component, that is defined as $\sigma_t^2 - q_t$ according to equation (2.5), which is mean-reverting towards the trend component q_t. Hence,

$$\sigma_t^2 = q_t + \alpha(\xi_{t-1}^2 - \sigma_{t-1}^2) + \beta(\sigma_{t-1}^2 - q_{t-1}) \quad (2.5)$$

$$q_t = \omega + \rho q_{t-1} + \phi \left(\xi_{t-1}^2 - \sigma_{t-1}^2\right) \quad (2.6)$$

with all five parameters restricted to be positive, $\alpha + \beta < \rho < 1$ and $\phi < \beta$. These parameter restrictions lead to several properties:

i) $\sigma_t^2 - q_t$ reverts to zero at an exponential decay rate of $\alpha + \beta$,

ii) the component q_t evolves following an AR(1) structure and converges to a constant level or daily unconditional variance, $\mathbb{E}^{\mathbb{P}}\left[\sigma_t^2\right]$, to be equal to $\omega/(1-\rho)$ in case of $\rho < 1$ and

[9]Some empirical studies are Bollerslev and Mikkelsen (1996, 1999), Taylor (2000), Stentoft (2005) and Christoffersen et al. (2008).

iii) the long-run component is more persistent since $\alpha + \beta < \rho$. When $\rho = 1$, we have the integrated C-GARCH process and we will analyze its impact on ESO pricing in Section 2.5.

2.2.2 Fair value of ESO

To obtain ESO prices in our discrete-time economy characterized in a GARCH context, we first apply the locally risk-neutral valuation relationship (LRNVR) in Duan (1995)[10] which is satisfied by a measure $\mathbb{Q}$ if

$$\mathbb{E}^{\mathbb{Q}}[S_t|\mathcal{F}_{t-1}] = S_{t-1}\exp(r-d) \tag{2.7}$$

$$Var^{\mathbb{Q}}\left[R_t|\mathcal{F}_{t-1}\right] = Var^{\mathbb{P}}\left[R_t|\mathcal{F}_{t-1}\right] = \sigma_t^2. \tag{2.8}$$

where the information set is now defined as $\mathcal{F}_{t-1} \equiv \{S_{t-i,i} \geq 1\}$. Given the standard Euler equation for asset pricing, that is obtained from an expected utility maximisation program, the validity for the LRNVR is held under several assumptions on the utility function and aggregate consumption. In short, LRNVR is verified under any of the following three conditions: (i) constant relative risk aversion (CRRA) utility function and changes in the logarithmic consumption distributed normally under $\mathbb{P}$–measure, (ii) constant absolute risk aversion (CARA) utility function and changes in the aggregate consumption distributed normally under $\mathbb{P}$–measure, and (iii) linear utility function[11]. Hence,

[10] The continuous-time economy associated with this discrete-time economy can be seen in Duan (1996). He shows that a more general GARCH process weakly converges to a certain bivariate diffusion process which nests some popular continuous-time stochastic volatility models for option valuation.

[11] See Theorem 1 in Duan (1995) for more details.

the LRNVR implies that the return dynamics evolve under the $\mathbb{Q}$-measure as

$$R_t = r - d - \frac{1}{2}\sigma_t^2 + \xi_t^*; \qquad \xi_t^* = \sigma_t z_t^*; \qquad z_t^* \sim i.i.d.\ N(0,1). \tag{2.9}$$

The GARCH and C-GARCH processes, driven by equations (2.2) and (2.5)-(2.6) respectively, become under this $\mathbb{Q}$–measure as

$$\sigma_t^2 = \omega + \alpha\sigma_{t-1}^2 \left(z_{t-1}^* - \lambda\right)^2 + \beta\sigma_{t-1}^2 \tag{2.10}$$

and

$$\sigma_t^2 = q_t + \alpha\sigma_{t-1}^2 \left(\left(z_{t-1}^* - \lambda\right)^2 - 1\right) + \beta\left(\sigma_{t-1}^2 - q_{t-1}\right) \tag{2.11}$$

$$q_t = \omega + \rho q_{t-1} + \phi\sigma_{t-1}^2 \left(\left(z_{t-1}^* - \lambda\right)^2 - 1\right). \tag{2.12}$$

Both conditional variance equations continue to have the same form except that the innovation is governed by a non-central chi-square random variable such that the non-centrality parameter equals the unit risk premium λ. This $\mathbb{Q}$-measure nests the conventional risk-neutralised pricing measure under homoskedasticity, i.e. $\alpha = \beta = 0$ for GARCH while $\rho = \phi = \alpha = \beta = 0$ for C-GARCH. That is, any of these restrictions on the variance parameters lead to the Black-Scholes framework[12]. It is also shown that the one-period ahead conditional variance is invariant with regards to a change to the risk-neutral measure $\mathbb{Q}$ while it does not hold beyond one period.

Of course any other alternative GARCH family specification with condition-

[12] This conventional measure is known as the risk-neutral valuation relationship (RNVR). For an excellent discussion about RNVR on asset pricing in a discrete-time economy, see the monograph of Poon and Stapleton (2005).

ally normal asset return innovations, the LRNVR can also be applied. However, some empirical studies, such as Bollerslev (1987), Baillie and Bollerslev (1989) and Hsieh (1989) reveal that the conditional normality for GARCH innovations may be questionable. This motivates the use of GARCH models with non-normal innovations. Two important distributional features of the GARCH innovations are conditional skewness and conditional heteroskedasticity. A generalized GARCH option pricing model, by allowing the asset return to exhibit a conditionally fat-tailed and skewed distribution, can be found in Duan (1999) and Stentoft (2007). The strategy for developing the generalized version of GARCH option valuation consists of transforming the conditional distribution of the innovations into conditional normality to apply the approach of Duan (1995). In short, the LRNVR under this new framework is referred as the generalized locally risk-neutral valuation relationship (GLRNVR)[13]. This methodology, that becomes rather cumbersome to implement in practice, is beyond the scope of this work but it is an interesting topic for future research.

Following Stentoft (2005), we simulate GARCH processes under the $\mathbb{Q}$-measure and apply the least-squares Monte Carlo (LSMC) approach to obtain the exercise rule across the paths as in Longstaff and Schwartz (2001). Stentoft (2005) shows that the LSMC algorithm is suitable to value American GARCH options since it produces very similar prices to those obtained with the lattice of Ritchken and Trevor (1999) and the Duan and Simonato (2001) method. We adapt this method to value a typical ESO characterized by an American call option with a maturity of T years, a vesting period of ν years, and a yearly exit rate or intensity

[13] An alternative to the LRNVR methodology is the approach of Siu et al. (2004), that is based on the concept of conditional Esscher transforms. It allows different infinitely divisible distributions for the GARCH innovations.

ε referred to a Poisson process.

We assume that the jump risk is not priced, that is, it can be diversified away. This assumption was first introduced by Merton (1976) with the important difference that in our work it is the option, but not the underlying asset, that is subject to jumps like in Jennergren and Näslund (1993). This assumption, which may be reasonable for the company issuing ESOs, is not realistic for the holder since the ESO is highly illiquid because it cannot be traded. This assumption is very common in a lot of works. See, among others, Jennergren and Näslund (1993), Carpenter (1998), Hull and White (2004) and Sircar and Xiong (2007).

The voluntary early exercise behavior is modeled as in Ammann and Seiz (2004). Their model adjusts the strike price —multiplying it by a factor m lower than one— in order to generate an early (suboptimal) exercise behavior. This approach provides similar results to models with several hard-to-estimate parameters, such as the utility-maximizing models, and it only needs to calibrate m with the expected life of the ESO[14]. Voluntary exercise occurs when the modified current payoff is larger than the discounted one period ahead expected value of the ESO, that is

$$(S_t - mK)^+ \geq e^{-r\delta t}\mathbb{E}_t^{\mathbb{Q}}\left[C_{t+\delta t}\right] \tag{2.13}$$

where the function $(y)^+$ is the maximum between y and 0 and $\mathbb{E}_t^{\mathbb{Q}}[\cdot]$ denotes the shortening of the conditional expectation. Note in equation (2.13) that if $m = 1$ we have the same exercise rule than in a tradable American option. If we set

[14] Ammann and Seiz (2004) also show that their model lead to rather the same ESO price as in the Hull and White (2004) tree if their respective early exercise parameters are calibrated so as to achieve the same expected life of the ESO.

$m < 1$, the left hand side of equation (2.13) will be larger and the voluntary exercise will occur earlier.

At time to maturity, the ESO is exercised if it is in the money and the value of the ESO is $C_T = (S_T - K)^+$. One period before, at $T - \delta t$ where δt is the length of a time step, on one hand there is a probability equal to $1 - e^{-\varepsilon \delta t}$ to abandon the firm —either voluntary or involuntary employment termination, that is assumed independent of the current stock price and time to maturity— and the payoff would be $(S_{T-\delta t} - K)^+$. On the other hand, the employee remains in the firm with a probability equal to $e^{-\varepsilon \delta t}$ and thus, he must decide either to hold or to exercise voluntarily the ESO. According to equation (2.13) he will exercise if $S_{T-\delta t} - mK > e^{-r\delta t}\mathbb{E}^{\mathbb{Q}}_{T-\delta t}[C_T]$, then the ESO value will be $S_{T-\delta t} - K$. Otherwise, the payoff will be the discounted expected value of the ESO. Thus, the ESO value at any time t verifying that $T > t \geq \nu$ is computed as

$$\begin{aligned} C_t &= e^{-\varepsilon \delta t}\left[X_t \mathbb{1}_{\{S_t - mK < X_t\}} + (S_t - K)^+ \mathbb{1}_{\{S_t - mK \geq X_t\}}\right] + \\ & \left(1 - e^{-\varepsilon \delta t}\right)(S_t - K)^+ \end{aligned}$$

where $X_t = e^{-r\delta t}\mathbb{E}^{\mathbb{Q}}_t[C_{t+1}]$ is the discounted risk-neutral expectation of the ESO value and $\mathbb{1}_{\{A\}}$ is an indicator funcition verifying that $\mathbb{1}_{\{A\}} = 1$ if A is true, but $\mathbb{1}_{\{A\}} = 0$ otherwise. The conditional expected value of the ESO will be computed by least-squares, that is, for those paths in the money, the one period ahead ESO value is regressed over some basis functions of both the current stock price and conditional volatility[15]. We work backwards until the vesting date with

[15] We have used both powers and Laguerre polynomials of current stock price, conditional volatility and their cross product as basis functions. We find that the prices are robust to the choice of the basis functions. This evidence is in accordance with Longstaff and Schwartz (2001) and Moreno and Navas (2003).

this scheme.

During the vesting period, the exercise is not allowed and the employee only can continue with the ESO, and if departure occurs the payoff would be zero. Hence, for any $t < \nu$ the option value is obtained recursively as

$$C_t = e^{-(r+\varepsilon)\delta t}\mathbb{E}_t^{\mathbb{Q}}\left[C_{t+1}\right] \tag{2.14}$$

Finally, the ESO fair value at current date is

$$C_0 = e^{-(r+\varepsilon)\nu}\mathbb{E}_0^{\mathbb{Q}}\left[C_\nu\right] \tag{2.15}$$

Note that this equation nests the European-style ESO price for $\nu = T$.

2.3 Numerical study

In this section we consider a hypothetical American ESO with a time to maturity of 10 years, no vesting restrictions and a continuously compounded dividend yield of 2.5%. The risk-free rate equals 5%, the starting price is 100 and the ESO is issued at the money (the usual way). We start pricing a hypothetical American ESO with constant volatility (CV) so as to compare it with our time-varying volatility framework. In the second stage, we value the same ESO but considering different GARCH models[16] and parameter sets.

Each price in the following tables and figures is the average of 100 estimates,

[16]We use the unconditional variance as the starting value in all simulations for GARCH models.

obtained with 50 different seeds plus the 50 antithetics. We allow 120 exercise points for the ESO life (monthly frequency), that is, we aproximate the ESO price using a Bermudan-style option[17] Each estimate has been obtained by running 20,000 paths. Simulated prices also verify the martingale property according to the empirical martingale simulation (EMS) of Duan and Simonato (1998). The EMS procedure ensures that the price satisfies the rational option pricing bounds. This method also yields a variance reduction particularly for at the money or longer-maturity options.

2.3.1 ESO pricing under constant volatility

Jennergren and Näslund (1993) present an early and easy model to value ESOs. This pricing model only adds one more parameter with respect to the option valuation model of Black and Scholes (1973). In the Jennergren and Näslund (1993) model, the ESO exercise/cancellation arrives with the first event of a Poisson process. This Poisson process is a proxy of the employee's early exercise or forfeiture comprising the wish of either portfolio diversification or consumption and also, the voluntary or involuntary employment termination. Using data on ESO exercises, Carpenter (1998) finds that the Jennergren and Näslund (1993) model predicts exercise times and ESO costs nearly the same as more sophisticated models such as the utility-maximizing ones, see Hall and Murphy (2002). Thus, ESO price in the Jennergren and Näslund (1993) model evolves according

[17] Stentoft (2004) obtains that the LSMC method with 10 exercise points per year produce very accurate prices compared with the ones obtained using a binomial model with 50,000 time steps. He argues that more accurate prices are obtained when increasing the simulated paths or the number of basis functions used as regressors.

the following PDE:

$$\frac{\partial C}{\partial t}+\frac{\partial C}{\partial S}(r-d)S+\frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2-rC+\lambda_e\left[\Psi(S,t)-C\right]=0 \tag{2.16}$$

such that $C\left(S;K,T\right)$ is the ESO price with S as the underlying stock price, K is the strike price and λ_e is the intensity per unit time (in years) of a Poisson process for an exercise event. Finally, $\Psi\left(S,t\right)$ is the holder's payoff if there is a jump event. Like ε in Section 2.2.2, the event risk driven by λ_e is not priced (see Section 2.2.2). For European ESO, $\Psi(S,t)=0$ for all t and the solution to equation (2.16) for the current date is

$$C(S;K,T)=e^{-\lambda_e T}\times BS\left(S,K,r,d,\sigma,T\right) \tag{2.17}$$

where $BS\left(S,K,r,d,\sigma,T\right)$ is the Black-Scholes formula with a constant continuously compounded dividend yield of d. Equation (2.17) simply says that the price of a European-style ESO is the ordinary BS price multiplied by the probability that the employee will remain in the firm until maturity. For American-style ESO, $\Psi\left(S,t\right)=\mathbb{1}_{\{T\geq t>v\}}(S-K)^+$ and its value is obtained by solving equation (2.16) numerically under a fully-implicit finite difference method. Specifically, to make the grid we set 2,520 steps for t (daily frequency) and 2,000 steps for S.

First, we value the hypothetical ESO assuming a constant yearly volatility of $\sigma=0.3$. To show the effects of the exit rate and the early exercise parameter, we implement a grid for both ε and m. Table 2.1 reports the price of our hypothetical ESO under different valuation methods. We consider two dividend policies: no dividend payments and a yearly continuously compounded dividend yield of $D = 0.025$. The case of $D = 0$ becomes interesting in order to isolate the possible early exercise decisions in the presence of dividends from the employee's diversification restrictions measured through $m < 1$. Note that standard American call options ($\varepsilon = 0$ and $m = 1$) would never be exercised in the absence of dividends. Columns JN_E and JN_A exhibit the prices for European and American-style ESO respectively, using the Jennergren and Näslund (1993) proposal. Finally, columns denoted as C_{cv} show the valuation of American ESOs with constant volatility using the simulation approach. Column E shows the corresponding European-style ESO prices using simulations. Below each price we report in parenthesis the price standard deviation.

According to equation (2.17) it is verified that $JN_E < BS$ for $\lambda_e > 0$ and $JN_E = BS$ for $\lambda_e = 0$ (i.e. $BS = 52.567$). In the absence of dividends (Panel A), note that JN_E and JN_A must also coincide for $\lambda_e = 0$. This negligible relative difference ($< 1\%$) between these two values is due to the error of the numerical method used to compute JN_A. For the case with dividends (Panel B), it is shown that $JN_A > JN_E$ as expected. Otherwise, $JN_A < BS$ for $\lambda_e > 0$ (i.e. $BS = 34.682$). This suggests that the well-known fact of BS overpricing also holds for the American-style ESO. Another result, in both panels, is the increasing value for the American ESO flexibility (measured as $JN_A - JN_E$) with higher values of λ_e. The more probable the cancellation/exercise event is, the higher the early exercise premium. Remember that λ_e column in Table 2.1 represents an intensity that captures both firm abandonment and early exercise for JN_A, while it does not occur for C_{cv} as we will explain right now.

The last three columns of Table 2.1, denoted as C_{cv}, display the prices using

Table 2.1: ESO price with constant volatility

λ_e/ε	JN_E	JN_A	C_{cv}			
			E	m		
				1.00	0.99	0.98
Panel A: $D = 0$						
0.00	52.567	52.553	52.547	52.655	51.684	49.141
			(0.11)	(0.60)	(0.91)	(1.58)
0.05	31.883	44.371	31.871	44.676	44.086	42.356
			(0.07)	(0.39)	(0.61)	(1.05)
0.10	19.338	38.295	19.331	38.601	38.216	37.091
			(0.04)	(0.30)	(0.43)	(0.70)
0.15	11.729	33.643	11.725	34.001	33.729	32.974
			(0.03)	(0.21)	(0.32)	(0.49)
Panel B: $D = 0.025$						
0.00	34.682	36.304	34.661	36.424	36.031	34.759
			(0.12)	(0.33)	(0.53)	(1.31)
0.05	21.035	31.618	21.023	31.823	31.563	30.668
			(0.07)	(0.24)	(0.32)	(0.78)
0.10	12.759	28.022	12.751	28.284	27.965	27.352
			(0.05)	(0.18)	(0.24)	(0.57)
0.15	7.738	25.211	7.734	25.458	25.319	24.808
			(0.03)	(0.15)	(0.19)	(0.40)

This table shows the price of an American-style ESO with a constant yearly volatility of $\sigma = 0.30$ for the underlying asset. Different valuation methods are reported. We select a time to maturity of $T = 10$ years and a yearly risk-free rate of $r_f = 0.05$. We consider two dividend policies: no dividend payments (Panel A) and a yearly continuously compounded dividend yield of $D = 0.025$ (Panel B). The column λ_e/ε shows the different intensities and exit rates for the JN and simulation model respectively. The columns JN_E and JN_A show the prices for a European and American ESO respectively according to the Jennergren and Näslund (1993) proposals. Finally, the last four columns report the ESO prices using simulations (C_{cv}) for the European case in column E and under different early exercise parameters (m) for the American-style ESO. In parenthesis, we display the price standard deviation.

the simulation approach —where the return, under the risk-neutral measure $\mathbb{Q}$, is driven by equation (2.9) but with constant volatility— for different early exercise parameters. The content of λ_e in JN is divided in two different components in C_{cv}. That is, the involuntary exercise or cancellation rate is measured by ε while the voluntary early exercise is measured through m[18]. As expected, C_{cv} (E) and $C_{cv}(m = 1)$ prices are quite similar to JN_E and JN_A respectively in both panels[19]. Note that JN_E, JN_A, C_{cv} (E) and C_{cv} ($m = 1$) are rather the same price for $\varepsilon = 0$ and $d = 0$. Finally, C_{cv} achives the highest value for $m = 1$ which is the case of an unconstrained agent.

2.3.2 ESO pricing under time varying volatility

It becomes a question of interest the possible misspricing when assuming erroneously a constant volatility model instead of a time-varying volatility one as the true process. To carry out this analysis we value our hypothetical ESO but taking different parameter sets for each GARCH model described in Section 2.2.1.

Tables 2.2 and 2.3 exhibit the prices under the two GARCH models considered. In parenthesis, we show the standard deviation of the estimated price. We assume that the market price of risk, λ, is equal to zero. It means that the unconditional variance coincides under the two measures $\mathbb{P}$ and $\mathbb{Q}$. In Section

[18]We have characterized this relationship more explicitly through a calibration exercise. In short, for each pair (m, ε), we obtain that value of λ_e, denoted as λ_e^*, satisfying $JN(\lambda_e^*) = C_{cv}(m, \varepsilon)$. For instance, (i) if we set for the pair (m, ε) the values of (0.99, 0.05) and (0.98, 0.05), then λ_e^* equals 0.0530 and 0.0667 respectively; (ii) meanwhile for (0.99, 0.10) and (0.99, 0.15), then λ_e^* equals 0.1014 and 0.1506 respectively. Note that both examples (i) and (ii) study the sensitivity analysis by keeping fixed one parameter, i.e. ε and m respectively.

[19]The small differences between prices are again due to the numerical valuation methods. The relative differences are lower than 1%.

2.4.2, we study the impact of λ in ESO pricing and its financial implications. We must point out that the CV model is nested in any of the GARCH models given some specific values for each GARCH parameter set since all imply a yearly unconditional volatility of 0.3.

Under the GARCH(1,1) model, the prices from panel A are higher than those in panel B and these are higher than those for panel C. Note that this effect occurs going from higher (lower) to lower (higher) values of β (α) with a constant persistence level, $\alpha + \beta$, of 0.99. For the C-GARCH prices, we increase across panels the persistence of the long-run component variance, i.e. $\rho = 0.99$, $\rho = 0.995$ and $\rho = 0.9975$, and we adjust ω to obtain the desired yearly volatility of 0.30. It is exhibited a decrease in the ESO price as ρ increases.

As we have comented in Section 2.2.1, GARCH models create leptokurtosis for the unconditional distribution under the $\mathbb{P}$-measure, and depending on the GARCH parameter set the excess of kurtosis may vary. Thus, it is interesting to answer the question about the possible relation between the excess of kurtosis implicit in the GARCH models and the results in Tables 2.2 and 2.3. We conduct the following exercise:

1. for each GARCH model and parameter set we simulate 10,000 log-return paths with a length of 2520 observations (ten years with daily frequency),

2. each month (each 21 observations) we estimate the sample kurtosis in cross-section over the 10,000 paths[20], and

[20]We only estimate the kurtosis each month because we apply the LSMC algorithm monthly, and it is enough to observe some regularities.

Table 2.2: ESO price under GARCH(1,1)

ε	E		$m = 1.00$		$m = 0.99$		$m = 0.98$	
Panel A: $\alpha = 0.1,\ \beta = 0.89$								
0.00	34.020	(0.70)	35.654	(1.14)	35.064	(1.33)	33.324	(2.87)
0.05	20.634	(0.42)	31.097	(0.76)	30.666	(0.95)	29.577	(1.81)
0.10	12.515	(0.26)	27.550	(0.56)	27.278	(0.70)	26.348	(1.07)
0.15	7.591	(0.16)	24.824	(0.43)	24.546	(0.65)	23.843	(1.23)
Panel B: $\alpha = 0.15,\ \beta = 0.84$								
0.00	32.789	(0.91)	34.569	(1.34)	33.747	(1.87)	31.529	(3.21)
0.05	19.887	(0.55)	30.134	(0.89)	29.534	(1.34)	27.845	(2.45)
0.10	12.062	(0.33)	26.708	(0.65)	26.264	(0.97)	24.981	(1.83)
0.15	7.316	(0.20)	24.016	(0.52)	23.664	(0.76)	22.763	(1.31)
Panel C: $\alpha = 0.2,\ \beta = 0.79$								
0.00	31.233	(0.82)	33.411	(1.18)	32.624	(1.51)	29.813	(4.21)
0.05	18.944	(0.50)	29.014	(0.91)	28.488	(1.14)	26.297	(3.42)
0.10	11.490	(0.30)	25.654	(0.75)	25.264	(0.90)	23.542	(2.88)
0.15	6.969	(0.18)	23.017	(0.69)	22.722	(0.77)	21.318	(2.33)

This table shows the price of an American-style ESO where the underlying asset is simulated under alternative GARCH(1,1) parameter sets in equation (2.2). Each parameter set supports an annualized unconditional expected volatility of 0.30 and a persistence level of $\alpha + \beta = 0.99$. The selected time to maturity of the ESO is $T = 10$ years, the risk-free interest rate is $r_f = 0.05$, the market price of risk is $\lambda = 0$ and the continuously compounded dividend yield is $D = 0.025$. The first column, ε, exhibits the different exit rates considered. Different early exercise parameter values are denoted by m.

Table 2.3: ESO price under C-GARCH

ε	E		$m = 1.00$		$m = 0.99$		$m = 0.98$	
Panel A: $\omega = 3.5714 \times 10^{-6},\ \rho = 0.99$								
0.00	34.102	(0.71)	36.041	(0.71)	35.563	(1.33)	33.908	(2.24)
0.05	20.690	(0.43)	31.422	(0.84)	31.069	(1.01)	29.974	(1.54)
0.10	12.549	(0.26)	27.876	(0.70)	27.607	(0.80)	26.638	(1.48)
0.15	7.612	(0.16)	25.074	(0.60)	24.874	(0.67)	24.267	(0.97)
Panel B: $\omega = 1.7857 \times 10^{-6},\ \rho = 0.995$								
0.00	33.751	(0.70)	35.336	(1.07)	34.770	(1.32)	33.038	(2.71)
0.05	20.476	(0.43)	30.838	(0.74)	30.411	(0.91)	29.169	(1.93)
0.10	12.419	(0.26)	27.352	(0.57)	27.051	(0.64)	26.120	(1.36)
0.15	7.533	(0.16)	24.616	(0.46)	24.391	(0.50)	23.661	(1.08)
Panel C: $\omega = 8.9286 \times 10^{-7},\ \rho = 0.9975$								
0.00	32.907	(0.31)	33.920	(1.91)	32.906	(2.60)	30.752	(3.37)
0.05	19.971	(0.50)	29.649	(1.36)	28.919	(1.95)	27.315	(2.63)
0.10	12.113	(0.31)	26.347	(1.10)	25.836	(1.40)	24.609	(1.97)
0.15	7.347	(0.19)	23.771	(0.83)	23.401	(1.07)	22.365	(1.56)

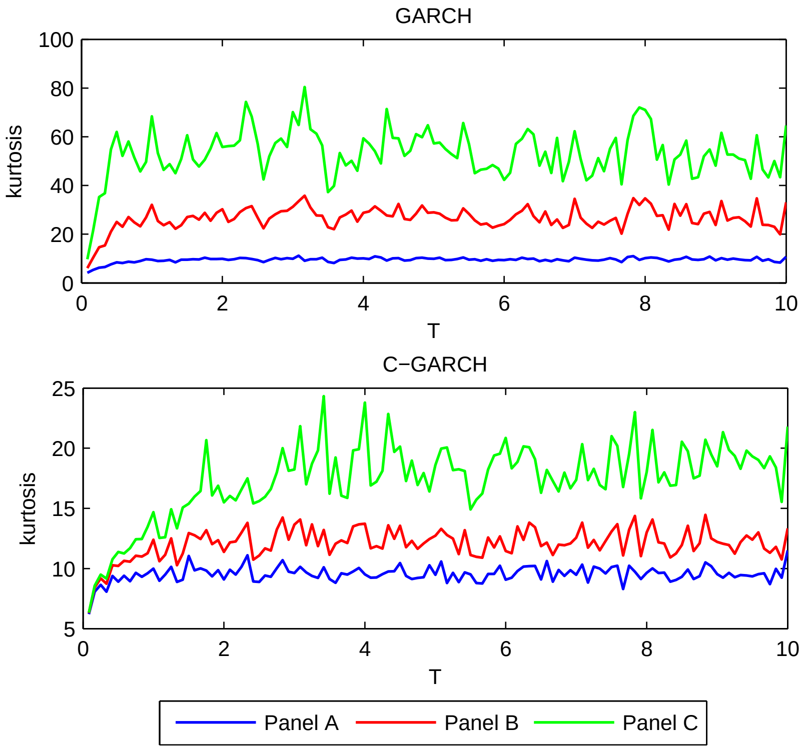
This table shows the price of an American-style ESO where the underlying asset is simulated under alternative GARCH parameter sets in equations (2.5) and (2.6). Each parameter set supports an annualized unconditional expected volatility of 0.30. The selected time to maturity of the ESO is $T = 10$ years, the risk-free interest rate is $r_f = 0.05$, the market price of risk is $\lambda = 0$ and the continuously compounded dividend yield is $D = 0.025$. The first column, ε, exhibits the different exit rates considered. Different early exercise parameter values are denoted by m.

3. we repeat the previous steps for 50 seeds plus the 50 antithetics.

Summarizing, for each GARCH model and parameter set we get 100 kurtosis for 120 months (10 years). Figure 2.1 shows the median of the kurtosis for the GARCH and C-GARCH under the parameter sets considered in Tables 2.2 and 2.3. It is shown that the kurtosis increases when moving from Panel A to C for both conditional volatility models. Specifically, although the GARCH persistence measured through $\alpha + \beta$ remains constant across the different panels, nevertheless the kurtosis increases for higher values of α but lower values of β to get the fixed value of 0.99 for the persistence. Hence, the kurtosis becomes very sensitive when changing the size for both parameters without changing the persistence level. Meanwhile, the kurtosis under the C-GARCH model increases with higher values for the persistence parameter ρ going from Panel A to C. Hence, given a GARCH model, there is a negative relation between the ESO price under a GARCH process and the kurtosis implicit in this process: the higher (lower) the kurtosis becomes, the lower(higher) the ESO price.

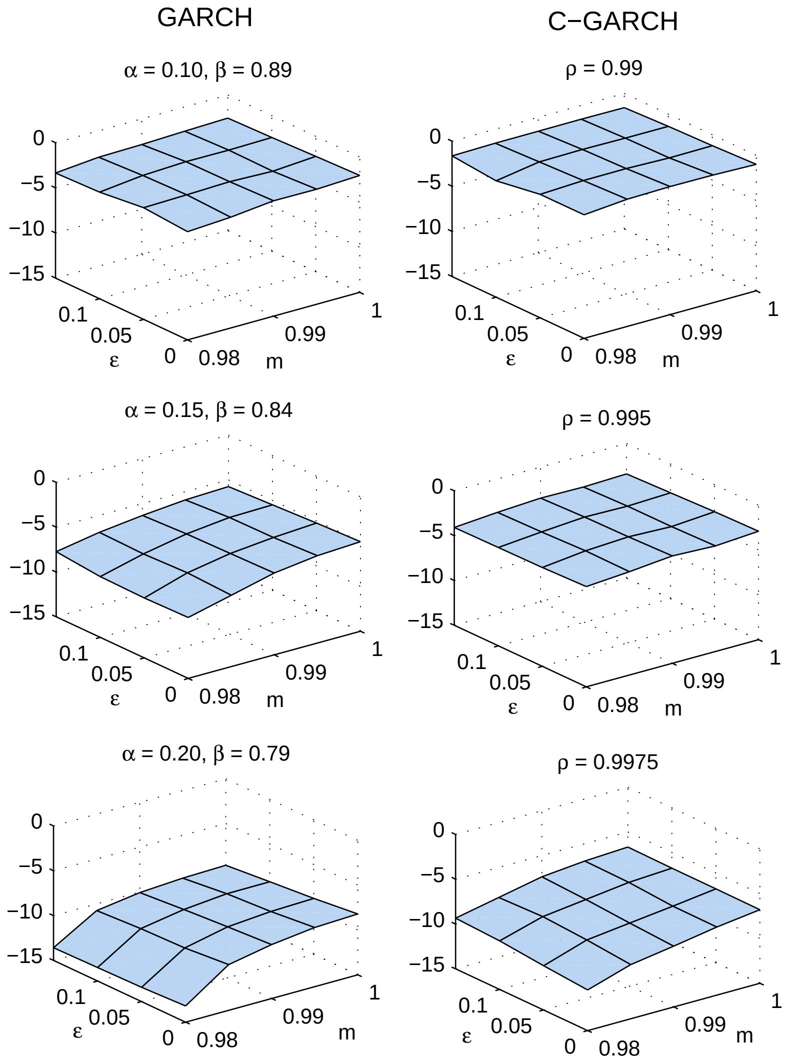
Figure 2.3 tries to explain why GARCH ESO prices are lower than CV ones. This figure displays the cumulative probability of exercise, $F(t)$, using the risk-neutral measure $\mathbb{Q}$. That is, $F(t) \equiv \mathbb{E}^{\mathbb{Q}}\left[\mathbb{1}_{\{A\}}\right]$ where $A \equiv \{Y \leq t\}$ such that Y represents the random variable "exercise date" with support $0 \leq t \leq T$. We only consider the case of $\varepsilon = 0.05$ and two alternative values of m: a suboptimal case ($m = 0.98$) and the optimal one ($m = 1$). Each picture exhibits $F(t)$ for all (C-)GARCH parameter sets in Table 2.2. Moreover, they include $F(t)$ under CV for comparative reasons (solid line). The pictures in the last row show the probability differences between the suboptimal case and the optimal one.

Figure 2.1: GARCH process and Kurtosis



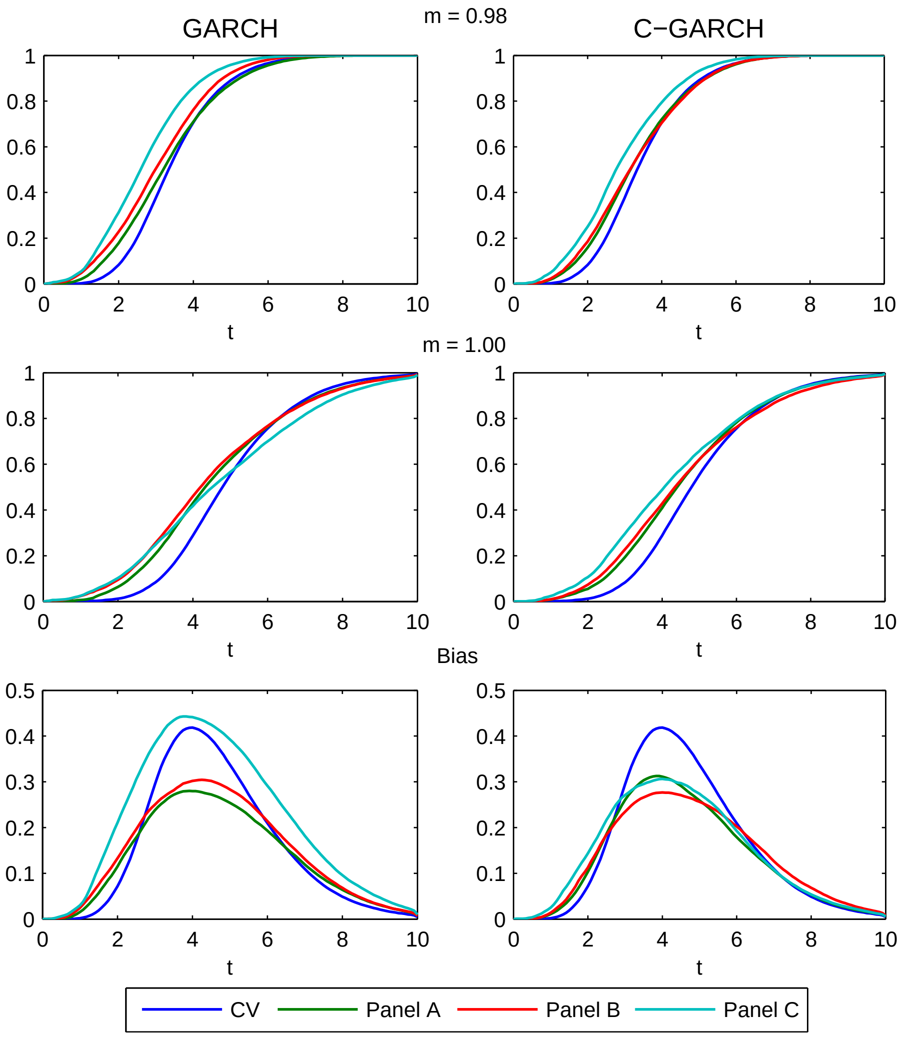
These graphics show the median of kurstosis obtained in cross section over the simulations of the GARCH models in Tables 2.2 and 2.3. Each simulations comprises 10,000 paths and each simulation has been repeated for 50 seeds and the 50 antithetics.

Figure 2.2: Price bias under different volatility models



These graphics show percentage relative biases of the American ESO prices for the two different GARCH models, compared with the American ESO price under constant volatility. The bias is computed as $\frac{C_g - C_{cv}}{C_{cv}} \times 100$ where C_g is the ESO price for each GARCH and C_{cv} is the ESO price for the constant volatility case reported in the last three columns of Table 2.1. A negative bias implies an undervaluation of the ESO under the GARCH framework. All cases have the same expected unconditional volatility. The parameters m and ε denote the early exercise rule and exit rate respectively.

Figure 2.3: ESO exercise probabilities



This figure shows the cumulative probability under the risk-neutral measure $\mathbb{Q}$ of ESO exercise, i.e. $F(t) \equiv \mathbb{E}^{\mathbb{Q}}[\mathbb{1}_A]$ where $A \equiv \{Y \leq t\}$ such that Y represents the random variable "exercise date" with support $0 \leq t \leq T$. The ESO characteristics are: $T = 10$, $r_f = 0.05$, $\lambda = 0$, $D = 0.025$ and $\varepsilon = 0.05$. The upper graphics hold an early exercise parameter of $m = 0.98$, while the middle ones hold the optimal exercise parameter, $m = 1$. The bottom graphics exhibit the probability differences between the suboptimal and the optimal cases. The GARCH parameter sets come from Tables 2.2 and 2.3.

We obtain the following results. First, we can always observe a positive difference (bias) suggesting that exercise occurs earlier for $m = 0.98$. This situation is in accordance with the behavior of a risk-averse and undiversified employee. This is the reason why ESOs are cheaper than tradable options ($m = 1$). Second, under the GARCH framework, $F(t)$ becomes higher, at least, during the first years of the option life. This fact suggests more early exercise decisions than in the CV setting. This explains why CV overprices, i.e. $C_{cv} > C_g$ in Figure 2.2, since its expected exercise time is higher. Third, note that F(t) increases with the persistence parameter ρ of the C-GARCH model during the first years of the option life. This is related to the behavior for the kurtosis just mentioned above. This suggests for an investor with restrictions ($m = 0.98$) to exercise earlier than in other situations with lower persistence, that is why C-GARCH ESO values become less expensive.

2.4 Sensitivity analysis

This section aims to analyze the impact on the ESO price by changing each time only one of the following parameters: the vesting period (ν), time to maturity (T) and the market price of risk (λ). For this analysis we only select the GARCH models with the parameter sets from panel C in Tables 2.2 and 2.3 for the case of $(\varepsilon, m) = (0.05, 0.98)$.

2.4.1 Vesting period and time to maturity

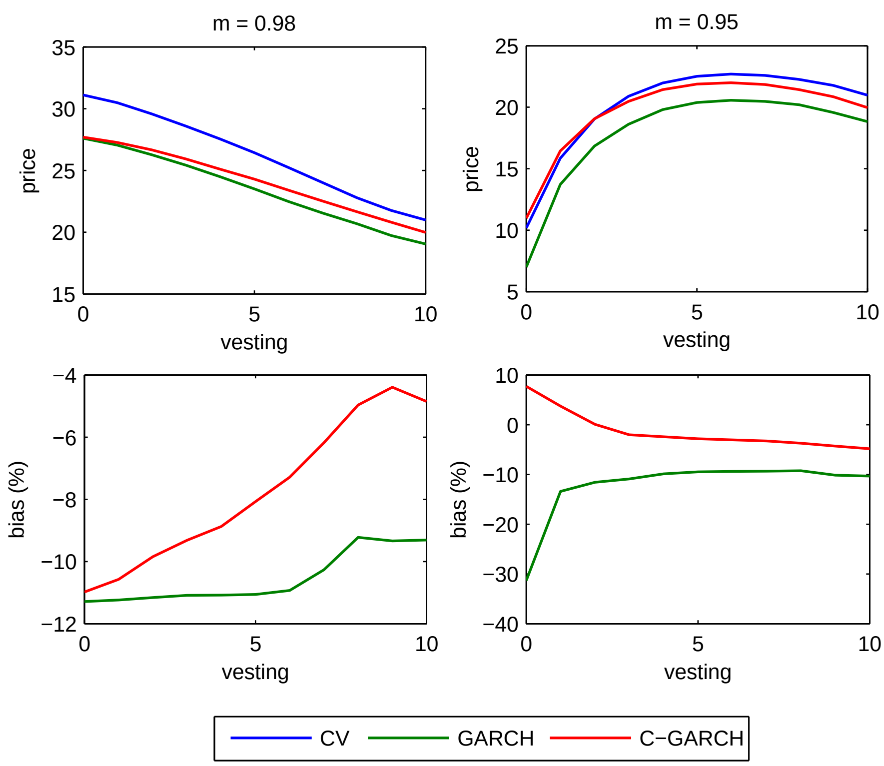
Figure 2.4 displays ESO prices for different vesting periods and two early exercise parameter m in the upper graphics. The left hand graphic holds the common situation of $m = 0.98$, while the right hand graphic is associated with a more suboptiomal exercise behaviour with $m = 0.95$. The bottom graphics exhibit the relative price biases in percentages, calculated with respect to the CV case. Note that, for $\nu = 10$ ($\nu = 0$) we have a European (fully American) ESO.

For the less suboptimal case ($m = 0.98$), we observe that the ESO cost is a decreasing function of the vesting period since for large vesting period the ESO becomes a European-style ESO. Moreover, the shorter (longer) the vesting period is, the larger (smaller) the size of the price bias. Note also that we would expect a decreasing behavior of the ESO price with the vesting period for the more suboptimal exercise parameter ($m = 0.95$). However, this case exhibits an inverted U-shape behavior. It suggests that ESO price increases for short vesting periods and then, it starts decreasing for larger ones. This phenomenon is due to the agent's restrictions leading to a suboptimal behavior. A long vesting period avoids the agent of accepting a lot of suboptimal situations that might be undertaken in case of either absence or a lower vesting period[21]. In contrast, for very large vesting periods, ESOs become more European, and hence prices decrease, regardless the vesting protection.

Concerning the time to maturity, a higher T implies a higher price as expected. If we assume erroneously a constant volatility model, then it always

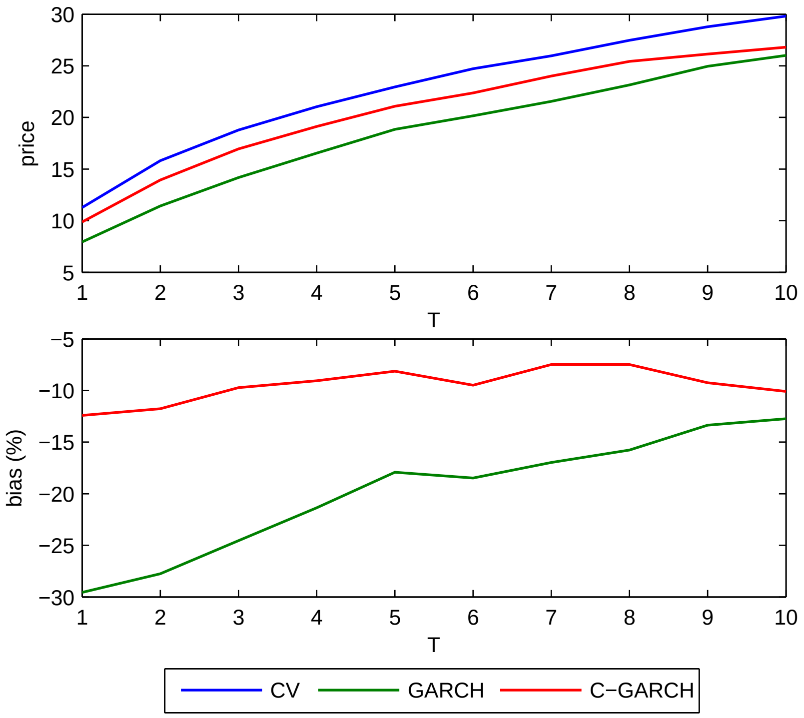
[21] Kulatilaka (2004) also points out the vesting protection phenomenon using a utility-based framework to determine the exercise rule.

Figure 2.4: ESO price and vesting period



This figure shows ESO prices for different vesting periods. The bottom graphic shows the percentage relative bias of the ESO price under alternative GARCH models (C_g) with respect to the constant volatility case, i.e. $\frac{C_g - C_{cv}}{C_{cv}} \times 100$. The ESO characteristics are: $S_0 = 100$, $r_f = 0.05$, $D = 0.025$, $\epsilon = 0.05$, $m = 0.98$ and $T = 10$. For the GARCH models, the parameter sets come from panel C in Table 2.2.

Figure 2.5: ESO price and time to maturity



This figure shows ESO prices for different time to maturities in the upper graphic while the bottom graphic shows the percentage relative bias of the ESO price under alternative GARCH models (C_g) with respect to the constant volatility case, i.e. $\frac{C_g - C_{cv}}{C_{cv}} \times 100$. The ESO characteristics are: $S_0 = 100$, $r_f = 0.05$, $D = 0.025$, $\varepsilon = 0.05$, $m = 0.98$ and $\nu = 0$. For the GARCH models, the parameter sets come from panel C in Tables 2.2 and 2.3.

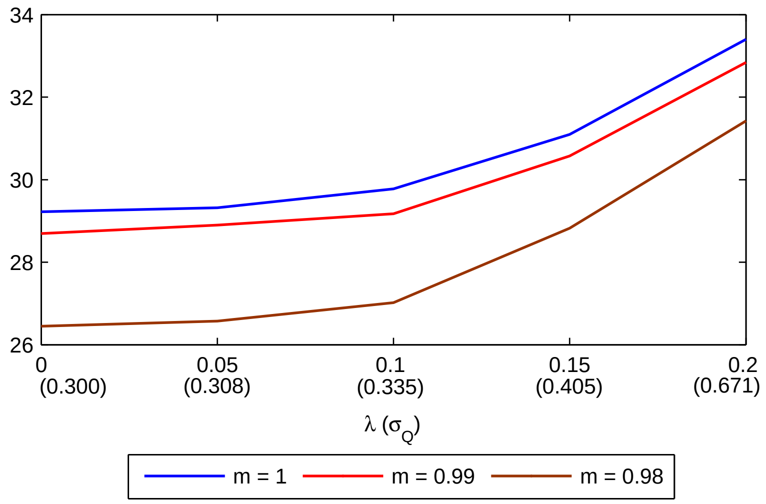
overprices and in the bottom graphic we can appreciate that the bias size decreases with the time to maturity.

2.4.2 The market price of risk

The unconditional variance under the risk-neutral measure, $\sigma^2_{\mathbb{Q}} \equiv \mathbb{E}^{\mathbb{Q}}\left[\sigma^2_t\right]$, is the relevant one for option pricing. It coincides with $\sigma^2 \equiv \mathbb{E}^{\mathbb{P}}\left[\sigma^2_t\right]$ under the CV model. However, $\sigma^2_{\mathbb{Q}}$ changes with λ under the GARCH family option pricing. In this case, the conditional variance under the $\mathbb{Q}$-measure reverts to an unconditional one higher than σ^2. Thus, the difference between GARCH and CV option prices can be regarded as using two different levels of the unconditional volatility. In short, the *BS* option price in the GARCH framework should be interpreted as using an incorrect unconditional standard deviation for the asset return under the risk-neutral measure $\mathbb{Q}$. That is, using σ^2 under the $\mathbb{Q}$-measure instead of $\sigma^2_{\mathbb{Q}}$. This fact leads to the following question: Does the CV ESO overvaluation in Section 2.3.2 hold when $\lambda \neq 0$?.

Figure 2.6 shows GARCH ESO prices for different values of λ under alternative values of m. We only select the GARCH model in panel C from Table 2.2 ($\omega = 3.5714 \times 10^{-6}$, $\alpha = 0.20$ and $\beta = 0.79$) with $\varepsilon = 0.05$. The x-axis displays different prices of risk with the corresponding unconditional yearly volatilities (in parentheses), according to $\sigma^2_{\mathbb{Q}} = \omega/(1 - \alpha\left(1 + \lambda^2\right) - \beta)$. Since $\partial\sigma^2_{\mathbb{Q}}/\partial\lambda > 0$, ESO price increases with λ when fixing m. If we look at Panel B in Table 2.1 and specifically, the row where $\varepsilon = 0.05$ then we can compare the ESO values denoted as C_{cv} with those in Figure 2.6 when $\lambda = 0$ for different values of m.

Figure 2.6: ESO price and the market price of risk



This figure shows the ESO price behavior as a function of the market price of risk (λ) for different early exercise parameters (m). The other parameters for ESO valuation are: $T = 10$, $r_f = 0.05$, $D = 0.025$ and $\varepsilon = 0.05$. The conditional variance of the underlying asset follows a GARCH process with parameters: $\omega = 3.5714 \times 10^{-6}$, $\alpha = 0.20$ and $\beta = 0.79$. In parenthesis, we display the corresponding yearly unconditional volatility implied in the GARCH model under the $\mathbb{Q}$-measure.

In this situation both valuations are driven by the same unconditional yearly volatility of 0.3, it holds that $C_g < C_{cv}$. If we increase λ, then C_g increases but it is necessary a very large value of λ to obtain $C_g > C_{cv}$. According to the empirical evidence[22], the daily estimation of this value becomes very small, of order 10^{-2}, suggesting the CV ESO overpricing.

[22] See Christoffersen and Jacobs (2004) and Stentoft (2005).

2.5 ESO pricing with a misspecified model

In this section we study the impact of pricing ESOs when the selected DGP for modeling the underlying stock price is not the right one and compare with the true process. Here, we analyze the sensitivity of ESO prices according to the degree of volatility memory, that is, C-GARCH (long-memory) against GARCH (short-memory).

For this study, we start with assuming the C-GARCH structure as the right process (true DGP) for the daily returns of the underlying stock price. The procedure to obtain ESO prices is as follows:

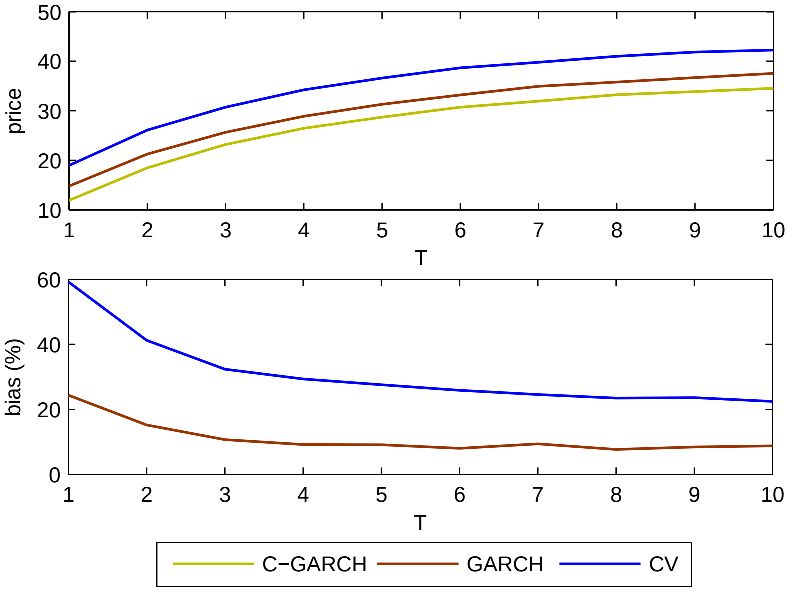
1. First, we simulate $1,000$ sample paths of length $3,000$ daily return observations each under the true DGP driven by an integrated C-GARCH model with the following parameters: $\lambda = 0$, $\omega = 8.9286 \times 10^{-7}$, $\alpha = 0.15$, $\beta = 0.80$, $\rho = 1.00$ and $\phi = 0.04$[23].

2. For each path, we leave out the first 500 observations to avoid the problem of starting values and undertake the maximum likelihood (ML) estimation assuming erroneously (false DGP) the GARCH specification for the remaining $2,500$ observations.

3. Third, we take the average over the $1,000$ estimates for each GARCH parameter. Finally, we use this mean vector as the GARCH parameter set for the ESO valuation exhibited in Figure 2.7 for different maturities. Specifically, the average parameters are: $\lambda = 0.0044$, $\omega = 1.886 \times 10^{-5}$, $\alpha = 0.1876$

[23]These parameters come from Panel C in Table 2.2 except the value of ρ.

Figure 2.7: ESO price under misspecified volatility models



This figure presents ESO prices for different time to maturities in years (T) when either GARCH or CV model is implemented by mistake (false DGP). We assume that C-GARCH model is the right process (true DGP). The procedure to obtain the ESO prices is described in Section 2.4. The parameters for ESO valuation are: $r_f = 0.05$, $D = 0.025$, $\varepsilon = 0.05$ and $m = 0.98$.

and $\beta = 0.7861$. This parameter set implies an unconditional yearly volatility of 0.4247.

We repeat the same experiment but starting at the second step where the incorrect DGP is now driven by the CV model that is also displayed. The average constant yearly volatility equals 0.4872. This figure exhibits the percentage relative biases, obtained respecting the true DGP (C-GARCH), for CV and GARCH ESO prices.

Some results emerge from Figure 2.7. First, there is a misspricing when assuming an incorrect volatility specification (either GARCH or CV) with respect to the correct one (C-GARCH). The size of this price difference becomes larger when assuming the CV model instead of the GARCH one. Both models overprice with respect to the C-GARCH process. This result is also in line with Ohanissian et al. (2004). They obtain that when the true DGP is driven by a true long-memory volatility process, then using as incorrect DGP either a short-memory model or a spurious long-memory one leads to general overpricing of European call options[24]. Second, both ESO price biases decrease as the time to maturity increases for short maturities, but they remain constant for longer maturities. In short, capturing the right memory volatility behavior becomes relevant in ESO pricing.

2.6 Accounting standards and ESO valuation

The FASB published the FAS 123 in 1995. This statement encourages, but does not oblige, firms to adopt a fair-value based method for accounting ESO expenses. It aims to obtain an approximation to the fair value through the BS formula, but replacing the ESO expiration date (T) by its expected exercise time or expected life (L), henceforth $BS(L)$, and correcting for the probability of departure during the vesting period. Thus, according to the FAS 123, the ESO price may be calculated as

$$C_{FAS,95} = BS(L) \times \exp(-\varepsilon\nu). \tag{2.18}$$

[24]They consider until two years as the time to maturity for their simulated option prices.

The principal advantage of FAS 123 method, or equation (2.18), is its simplicity since it is a closed-form formula, instead of numerical methods like finite differences or simulations. An interesting question is to compare our simulation-based GARCH model with the FAS 123 proposal. For this analysis, we plug the expected life obtained under simulation for different GARCH models into equation (2.18). If we consider the GARCH ESO expected life under this setting, we propose to modify equation (2.18) changing the *BS* price by the equivalent European GARCH option price in the spirit of FAS 123, that is

$$C_{FAS,g} = C_g^E(L) \times \exp(-\varepsilon\nu) \tag{2.19}$$

where $C_g^E(L)$ represents the European-style ESO price, with L as time to maturity, nested in the American ESO price (C_g) modeled in Section 2.2.2 when $T = \nu = L$.

The aim of Figure 2.8 is to compare both approximations, that is, FAS 123 formula (dashed line) and our European GARCH proposal (dotted line) with the correct value C_g (solid line) for different vesting periods (left hand graphics) and time to maturities[25] (right hand graphics). Note that each pair of prices $C_{FAS,95}$ and $C_{FAS,g}$ in any graphic of this figure holds the same time to maturity L. We would like to remark that $C_{FAS,g}$ equals C_g in two cases: $\nu = 10$ and $T = 3$. In these situations, C_g becomes a European-style ESO with $T = 10$ and $T = 3$ respectively. If we assume as correct DGP the constant volatility model (top graphics), we observe that $C_{FAS,95}$ and $C_{FAS,g}$ coincide since C_g^E in equation (2.19) becomes the *BS* price. Both approximations produce very similar prices

[25] In this case, we consider $\nu = 3$ years.

to the correct ones (C_g) independently of the time to maturity or the vesting period length. However, assuming the GARCH framework as the true DGP (middle and bottom graphics) $C_{FAS,95}$ overprices ESO for all ν and T considered. Meanwhile, our European-style proposal $C_{FAS,g}$ closely approximates to the true price C_g. In short, using European-style formulae to approximate the ESO price, the GARCH effects become relevant.

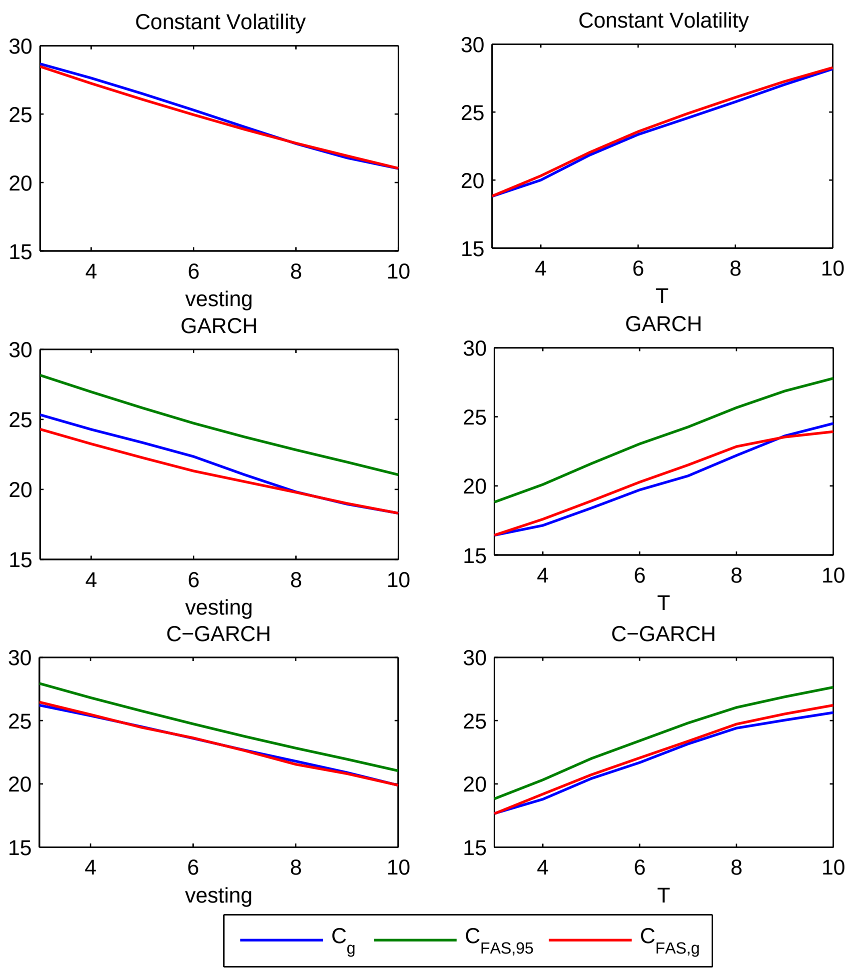
2.7 A real case study

In this section we value a real ESO plan providing some GARCH estimates and statistics. Moreover we show how to obtain the price confidence intervals through the asymptotic ESO price distribution.

2.7.1 Asymptotic ESO price distribution

Since ESOs are not tradable derivatives, we cannot estimate the vector of unknown true parameters ψ implied in the model for the underlying asset dynamics in the usual way of minimizing the mean squared option valuation error. Here, using the quasi-maximum likelihood (QML) criterion we estimate the parameters, denoted as $\widehat{\psi}$, from a discrete-time log-return series of an asset price. In order to show the error introduced into the valuation process by the need to estimate GARCH parameters, we obtain the asymptotic distribution of the QML ESO price estimator and we build the price confidence intervals. Let $\sqrt{n}(\widehat{\psi} - \psi) \overset{a}{\sim} N\left(0, V_{\psi}\right)$ where n is the sample size, V_{ψ} is the asymptotic

Figure 2.8: ESO price and FAS 123 approximation



These graphics show the true ESO price (C_g) using the simulation method (solid line), the equivalent FAS 123 price (dashed line) and the modified FAS 123 price (dotted line) under the different volatility models, and a range of vesting periods and time to maturities. The price under FAS 123 is $C_{FAS,95} = BS(L) \times e^{-\varepsilon v}$, where $BS(L)$ is the Black-Scholes price where the time to maturity is the ESO expected life obtained with the simulation method. The price under modified FAS 123 is $C_{FAS,g} = C_g^E(L) \times \exp(-\varepsilon\, v)$, where C_g^E denotes the European-style ESO price nested in C_g when $T = v = L$. For the right hand graphics we consider $v = 3$ years. The remaining parameters for ESO valuation are the same as those in Figure 2.5.

variance and consider a nonlinear function of ψ like the ESO price, denoted as $C \equiv C(\psi)$. If we apply the *delta method*[26] to a first-order Taylor series expansion of $C(\psi)$ around ψ[27], we obtain the asymptotic distribution of the QML ESO price estimator, denoted as $\widehat{C} \equiv C(\widehat{\psi})$, given by

$$\sqrt{n}(\widehat{C} - C) \overset{a}{\sim} N(0, V_c), \qquad V_c \equiv \frac{\partial C(\psi)'}{\partial \psi} V_\psi \frac{\partial C(\psi)}{\partial \psi} \tag{2.20}$$

where V_c may be estimated in the usual way

$$\widehat{V}_c \equiv V_c(\widehat{\psi}) = \frac{\partial C(\widehat{\psi})'}{\partial \psi} \widehat{V}_\psi \frac{\partial C(\widehat{\psi})}{\partial \psi} \tag{2.21}$$

such that $\widehat{V}_\psi$ is the asymptotic variance estimator. Therefore, for large n the variance of $\widehat{C}$ may be approximated by $Var(\widehat{C}) \approx V_c/n$.

2.7.2 ACS ESO plan and variance estimates

We compute both the fair value and confidence interval of the ESO plan granted by ACS[28] on May 2, 2004. The ACS stock price was trading at 13.91 euros on the grant date. The ESO plan consists on 7,038,000 options issued at the money with a time to maturity of six years. Specifically, one third of the options has a vesting period of three years, another third part has a vesting period of four years and the last part has a vesting period of five years.

[26]See Lo (1986) for more details.

[27]Higher order terms converge to zero faster than $1/\sqrt{n}$, that is why only the first term of the expansion matters for the asymptotic distribution of $C(\widehat{\psi})$. For more details, see Campbell et al. (1997).

[28]ACS is a construction industry Spanish firm and it belongs to the Spanish stock index IBEX-35.

Table 2.4: ACS Descriptive Statistics

mean ($\times100$)	0.0429	skewness	0.2914	
median ($\times100$)	0.0000	kurtosis	6.0676	
maximum	0.1379	JB	641.880	[0.000]
minimum	-0.0841	$Q(20)$	36.646	[0.013]
std. deviation (yearly)	0.3223	$Q^2(20)$	362.820	[0.000]

This table shows some descriptive statistics for the ACS daily stock return series for the period January 5, 1998 to April 30, 2004. JB is the Jarque-Bera statistic. $Q(20)$ and $Q^2(20)$ denote de Ljung-Box statistics for the return series and its squared ones respectively. In brackets, we show the corresponding p-values.

We use a daily time series of ACS returns, denoted as R_t, from January 2, 1998 to April 30, 2004 ($1,580$ observations) to estimate the CV and the GARCH models of Section 2.2.1. Table 2.4 displays some descriptive statistics for the return series. We can see that the R_t series shows both excess of kurtosis and positive skewness. As a result, the Jarque-Bera (JB) test rejects the null hypothesis of normality. Moreover, the Ljung-Box test statistic for the squared return series, denoted as Q^2 (20), clearly rejects the null hypothesis of independence, suggesting the existence of a time-varying variance dynamics and hence, modeling the returns according to a member from the GARCH family may be appropiate.

Figure 2.9 displays the time series and the sample autocorrelation function (ACF), up to lag 100, for the series R_t, R_t^2 and $|R_t|$. For the squared return series, we can observe the clustering phenomenon already described in Section 2.2.1. Note that most of the significant correlations for R_t^2 occur approximately until lag 40. Meanwhile, the ACF of the absolute returns shows evidence of a long-memory pattern since the correlations remain significant up to lag 80. This evidence also leads to the possibility of modeling under the C-GARCH structure.

Table 2.5: GARCH model estimates for ACS return series

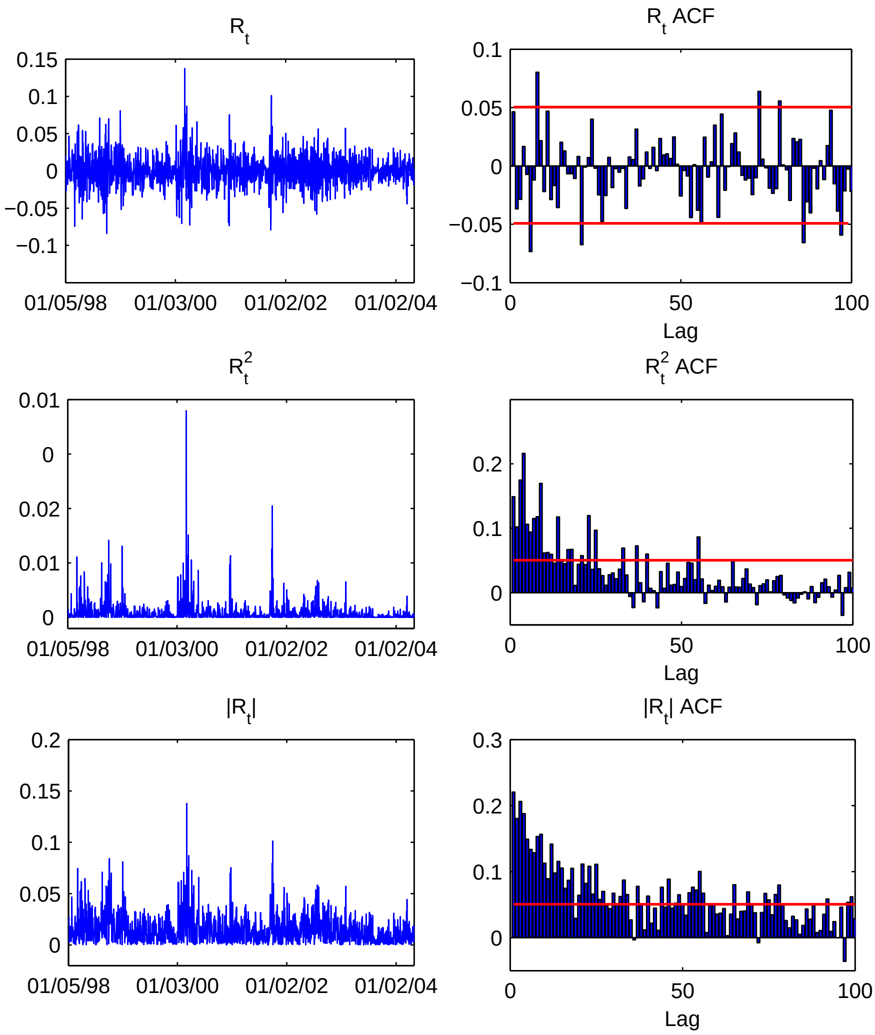
	CV		GARCH		C-GARCH	
	estim.	std.err.	estim.	std.err.	estim.	std.err.
λ	0.0234	(0.0252)	0.0238	(0.0238)	0.0252	(0.0241)
$\omega(\times 10^{-6})$	412.03*	(14.6887)	13.232*	(3.6166)	0.9172	(1.2754)
α	–		0.1354*	(0.0214)	0.1236*	(0.0227)
β	–		0.8371*	(0.0238)	0.7883*	(0.0459)
ρ	–		–		0.9973*	(0.0027)
ϕ	–		–		0.0241	(0.0184)
$\mathcal{L}(\hat{\psi})$	3915.66		4064.01		4070.15	
$\sqrt{\mathbb{E}^{\mathbb{P}}[\sigma_t^2]}$	0.3222		0.3482		0.2926	

This table shows the QML estimates for the GARCH models described in equations (2.2) and (2.5) – (2.6) respectively for the ACS daily stock return series for the period January 5, 1998 to April 30, 2004. In parenthesis, we display the standard error. The row $\mathcal{L}(\hat{\psi})$ contains the log-likelihood of the estimations. $\sqrt{\mathbb{E}[\sigma_t^2]}$ are the unconditional (yearly) volatilities obtained with the estimated variance parameters. The symbol * denotes significance levels at 1 %.

The QML estimates, in Table 2.5, show that the GARCH model is significant with a persistence level of 0.97. Meanwhile, the persistence measured through ρ is 0.997 for the C-GARCH. The row denoted as $\sqrt{\mathbb{E}^{\mathbb{P}}[\sigma_t^2]}$ collects the square root of the ($\mathbb{P}$-measure) unconditional variances per year implicit in the estimated parameters from the different conditional variance models. Note that the GARCH model yields the maximum unconditional yearly volatility, meanwhile the C-GARCH the minimum one.

If we analyze the goodness of fit, in Table 2.6, we conclude that the C-GARCH makes the best performance according to the Akaike information criterion (AIC).

Figure 2.9: Time series and autocorrelation plots of ACS



The left hand side exhibits the time series plots of ACS daily returns (R_t), its squared returns (R_t^2) and its returns in absolute value ($|R_t|$). Meanwhile, the right hand side exhibits the corresponding plots of the autocorrelations (ACF) for the first 100 lags with the confidence intervals ($\pm 1.96 \times n^{-1/2}$ where n is the sample size). The holding period goes from January 5, 1998 to April 30, 2004 (1,580 observations).

Table 2.6: Goodness of Fit of GARCH models for ACS return series

	CV	GARCH	C-GARCH
Akaike	-7827.32	-8120.01	$-8128.29\dagger$
BIC	-7816.59	$-8098.55\dagger$	-8096.10
SIC	-4.9472	$-5.1257\dagger$	-5.1241
JB_{res}	641.0669 [0.0000]	122.2564 [0.0000]	138.2324 [0.0000]
$Q_{res}(20)$	36.64147 [0.0129]	26.7789 [0.1415]	25.9027 [0.1690]
$Q^2_{res}(20)$	364.7673 [0.0000]	15.4429 [0.7505]	16.8783 [0.6608]

This table shows some goodness of fit statistics for the GARCH models estimated in Table 2.7.2 over ACS daily stock return series for the period January 5, 1998 to April 30, 2004. The three rows named AIC, BIC and SIC report the Akaike, Bayesian and Schwarz information criteria respectively. The symbol † denotes the best model according to the selected statistic criterion. The JB_{res} row exhibits the Jarque-Bera statistics for the estimated residuals and the corresponding p-values. $Q_{res}(20)$ and $Q^2_{res}(20)$ denote de Ljung-Box statistics for the GARCH standarized residuals. In brackets, we show the correspondig p-values.

Nevertheless, if we select under either Bayesian (BIC) or Schwarz (SIC) criterion —both penalize the number of the estimated parameters stronger than AIC— then GARCH and C-GARCH score rather similar. Finally, the statistic $Q^2_{res}\,(20)$ for the squared of the standardised residuals clearly cannot reject the null hypothesis of residual independence for any GARCH model but not for the CV case. This suggests that such conditional heteroskedastic variance structure is necessary for modeling R_t. We also obtain the same conclusion for the autocorrelation of the standardised residuals, that is, $Q_{res}(20)$ never leads to reject the null hypothesis except for the CV case.

2.7.3 ACS ESO price and confidence intervals

Tables 2.7, 2.8 and 2.9 display the estimates of the ESO price and the 95% asymptotic confidence intervals for the mean price (C) under different volatility specifications, according to equations (2.20) and (2.21). For each variance model, we also study the sensitivity of the ESO price estimator, $\widehat{V}_c/n$, and the confidence interval ($I_{95\%}$)[29] for alternative values of the vesting period, departure rate and the early exercise parameter. In Table 2.7 $\sqrt{\widehat{V}_c/n}$ has been multiplied by a factor equal to 10^4, meanwhile in Tables 2.8 and 2.9 the factor is 10^2, i.e. for the GARCH(1,1) model, $m = 0.99$, $\nu = 3$ and $\varepsilon = 0.05$ the true standard deviation of the estimated ESO price is 1.20×10^{-2}. The derivative $\partial C(\widehat{\psi})/\partial\psi$ in equation (2.21) does not show a closed-form solution. It suggests that the derivative must be obtained numerically by perturbating the value of $\widehat{\psi}$ slightly and then, obtaining the approximation of $\Delta C(\widehat{\psi})/\Delta\psi$[30].

For any volatility model, it holds that $\widehat{C}$ decreases with a larger value of ε but increases with m. Of course, these results were already expected according to the analysis made in Sections 2.3.1 and 2.3.2. It is verified that $\widehat{C}$ under GARCH (CV) leads to a higher value than CV (C-GARCH) in contrast with the finding in Section 2.3.2. This result occurs because $\sqrt{\mathbb{E}^{\mathbb{P}}[\sigma_t^2]}$ under GARCH (CV) is higher than CV (C-GARCH). Thus, the choice of an incorrect conditional variance model also may create a bias due to its ability to capture the true unconditional variance. Respect to the standard error of price estimates, we obtain the following results. First, the standard errors for the CV model become much

[29] A second-order Taylor series expansion would require the Hessian matrix that would be very computationally intensive. For instance, it is necessary to calculate a total of 21 numerical derivatives to estimate the new asymptotic standard errors for C-GARCH ESO prices.

[30] See Glasserman (2003) for more details.

smaller than in the GARCH framework. The size of the CV standard errors is 10^{-2} times the (C-)GARCH ones. Second, for $\nu = 4$ and $\nu = 5$ they decrease as ε increases for all variance models. However, for $\nu = 3$ this result holds in a few situations. Thus we can conclude that, for large vesting periods, price confidence intervals are narrower as larger is the exit rate ε.

Table 2.7: ACS ESO plan: prices and confidence intervals under Constant Volatility

		$m = 1.00$			$m = 0.99$			$m = 0.98$		
ν	ε	$\widehat{C}$	$\sqrt{\widehat{V}_c/n}$	$I_{95\%}$	$\widehat{C}$	$\sqrt{\widehat{V}_c/n}$	$I_{95\%}$	$\widehat{C}$	$\sqrt{\widehat{V}_c/n}$	$I_{95\%}$
3	0.05	3.571	1.51	(3.571, 3.572)	3.541	1.20	(3.540, 3.541)	3.406	1.17	(3.406, 3.407)
	0.10	3.015	1.20	(3.014, 3.015)	2.990	1.37	(2.990, 2.991)	2.880	1.18	(2.880, 2.881)
4	0.05	3.409	1.69	(3.409, 3.409)	3.385	1.58	(3.385, 3.385)	3.262	1.21	(3.262, 3.262)
	0.10	2.761	1.22	(2.760, 2.761)	2.743	1.25	(2.742, 2.743)	2.644	0.99	(2.644, 2.645)
5	0.05	3.233	1.33	(3.233, 3.233)	3.212	1.17	(3.212, 3.213)	3.120	1.19	(3.120, 3.120)
	0.10	2.503	0.92	(2.503, 2.503)	2.488	0.93	(2.488, 2.488)	2.417	0.95	(2.417, 2.417)

This table shows both the QML estimated prices ($\widehat{C}$) and their corresponding standard deviations $\left(\sqrt{\widehat{V}_c/n}\right)$ of the three different ESOs, each depending on a different vesting period (ν) in the first column, of the ACS plan granted on May 2, 2004. All ESOs have a time to maturity of six years. The initial price is 13.91 euros and ESOs are issued at the money. Each panel holds a different volatility model and contains the valuation of the same ESO for different early exercise parameters (m) and exit rates (ε).The parameters for the different GARCH models are those from Table 2.7.2. $I_{95\%}$ denotes the 95% confidence interval for the population mean of ESO price (C).

Table 2.8: ACS ESO plan: prices and confidence intervals under GARCH(1,1)

		$m = 1.00$			$m = 0.99$			$m = 0.98$		
ν	ε	$\widehat{C}$	$\sqrt{\widehat{V}_c/n}$	$I_{95\%}$	$\widehat{C}$	$\sqrt{\widehat{V}_c/n}$	$I_{95\%}$	$\widehat{C}$	$\sqrt{\widehat{V}_c/n}$	$I_{95\%}$
3	0.05	3.772	1.04	(3.751, 3.792)	3.741	1.10	(3.720, 3.763)	3.626	0.90	(3.609, 3.644)
	0.10	3.185	1.01	(3.165, 3.205)	3.161	1.55	(3.130, 3.191)	3.069	0.98	(3.050, 3.089)
4	0.05	3.600	0.85	(3.583, 3.617)	3.573	1.00	(3.553, 3.592)	3.471	1.25	(3.446, 3.495)
	0.10	2.916	0.70	(2.902, 2.930)	2.895	0.82	(2.879, 2.911)	2.815	0.82	(2.799, 2.831)
5	0.05	3.412	0.86	(3.395, 3.429)	3.387	0.84	(3.371, 3.403)	3.319	0.77	(3.304, 3.334)
	0.10	2.642	0.63	(2.629, 2.654)	2.623	0.63	(2.611, 2.636)	2.571	0.58	(2.559, 2.582)

This table shows both the QML estimated prices ($\widehat{C}$) and their corresponding standard deviations $\left(\sqrt{\widehat{V}_c/n}\right)$ of the three different ESOs, each depending on a different vesting period (ν) in the first column, of the ACS plan granted on May 2, 2004. All ESOs have a time to maturity of six years. The initial price is 13.91 euros and ESOs are issued at the money. Each panel holds a different volatility model and contains the valuation of the same ESO for different early exercise parameters (m) and exit rates (ε).The parameters for the different GARCH models are those from Table 2.7.2. $I_{95\%}$ denotes the 95% confidence interval for the population mean of ESO price (C).

Table 2.9: ESO plan: prices and confidence intervals under Component-GARCH

		$m = 1.00$			$m = 0.99$			$m = 0.98$		
ν	ε	$\widehat{C}$	$\sqrt{\widehat{V}_c/n}$	$I_{95\%}$	$\widehat{C}$	$\sqrt{\widehat{V}_c/n}$	$I_{95\%}$	$\widehat{C}$	$\sqrt{\widehat{V}_c/n}$	$I_{95\%}$
3	0.05	3.237	0.50	(3.227, 3.247)	3.188	0.18	(3.185, 3.192)	3.022	0.38	(3.014, 3.029)
	0.10	2.733	0.65	(2.720, 2.745)	2.694	0.58	(2.683, 2.701)	2.563	0.12	(2.561, 2.565)
4	0.05	3.092	0.34	(3.085, 3.099)	3.049	0.24	(3.044, 3.054)	2.901	0.71	(2.892, 2.919)
	0.10	2.504	0.10	(2.503, 2.506)	2.472	0.09	(2.470, 2.473)	2.359	0.14	(2.357, 2.362)
5	0.05	2.938	1.49	(2.909, 2.967)	2.906	0.51	(2.896, 2.916)	2.822	0.13	(2.820, 2.825)
	0.10	2.275	0.70	(2.261, 2.288)	2.251	0.53	(2.240, 2.261)	2.187	0.00	(2.187, 2.187)

This table shows both the QML estimated prices ($\widehat{C}$) and their corresponding standard deviations $\left(\sqrt{\widehat{V}_c/n}\right)$ of the three different ESOs, each depending on a different vesting period (ν) in the first column, of the ACS plan granted on May 2, 2004. All ESOs have a time to maturity of six years. The initial price is 13.91 euros and ESOs are issued at the money. Each panel holds a different volatility model and contains the valuation of the same ESO for different early exercise parameters (m) and exit rates (ε).The parameters for the different GARCH models are those from Table 2.7.2. $I_{95\%}$ denotes the 95% confidence interval for the population mean of ESO price (C).

2.8 Concluding Remarks and Extensions

This paper presents an application of the well-known LSMC technique for the valuation of ESO grants when the underlying asset follows a GARCH family structure. We compare this option value with the constant volatility situation, which becomes less descriptive of actual stock return dynamics than GARCH according to the empirical evidence. The principal finding is the ESO overpricing when a constant volatility model is assumed erroneously instead of a GARCH family model as the true process. This overpricing is reinforced when a long-memory volatility model (C-GARCH) is assumed as the true DGP but we have assumed as an incorrect DGP either the short-memory volatility model (GARCH) or the constant volatility one. Hence, alternative volatility models lead to different exercise strategies for the same ESO holder who is restricted to a certain level of either liquidity or diversification. Of course, the size of this ESO mispricing also depends on other parameters such as the life of the option (T), the vesting period (ν), the exit rate (ε), the market price of risk (λ) and the degree for the suboptimal exercise (m). These conclusions are based on numerical simulations since no closed formulae are available for this discrete-time environment of American ESOs with time varying volatility for stock returns.

Measuring this mispricing becomes relevant for the accounting standards. A modified Black-Scholes formula, which assumes a constant volatility model, is the benchmark model for ESO pricing since it was initially introduced by the FAS 123 in 1995. Note also that most ESOs are of American-style and hence, they are approximated using the BS formula by inserting the expected life parameter L instead of T and considering the departure risk. A correction to the benchmark European ESO valuation is proposed here and it works pretty well when comparing to the true ESO price defined as an American GARCH option. Indeed, we introduce a European GARCH ESO formula instead of the *BS* formula and using the expected life parameter.

Some important extensions for a future research are the following suggestions. First, since the two important ESO parameters ε and m have been assumed constant throughout this work, it would be very interesting to extend our American GARCH ESO valuation to endogenous patterns for both parameters. For instance, the probability of the employee's leaving the firm might be related to the stock price level through the moneyness ratio as in Carr and Linetsky (2001). Second, another possible implementation would be the valuation of indexed ESOs under this new context. See Johnson and Tian (2000) for the European case under constant volatility. Third, the impact on the ESO valuation of non-normal innovations for the conditional volatility. This extension leads to a more general locally risk-neutral valuation relationship that was introduced by Duan (1999). Last but not least, the possibility of pricing the jump risk for the ESO. Finally, obtaining finite sample confidence intervals for American ESO prices when the conditional variance parameters have been obtained by maximum likelihood as in Section 2.7. Note that the finite sample standard errors become preferable to the asymptotic ones based on the delta method to compute the ESO prices. See Dotsis and Markellos (2007) for the easier case of pricing European GARCH options.

3

Subjective Executive Stock Option Valuation: the One State-Variable Framework

3.1 Introduction

Executive stock options (ESOs hereafter) have been widely used by US and EU firms during the last two decades. Hall and Murphy (2002) reports that in 1999, 94 % of S&P 500 companies granted ESOs to their top executives, and these grants accounted for 47 % of total pay for S&P CEOs. The main reason to use ESOs in compensation packages is to mitigate the principal-agent problem, since ESOs relate the executive's wealth with the company stock price. Besides the ESOs, executives usually are also granted with restricted stock of the company. In consequence, the executive's portfolio supports more firm specific risk than a well diversified investor. Moreover, insiders are not allowed to sell their firm's stock short by the Section 16-C of the Securities Exchange Act. This lack of diversification may affect the incentive and cost of the ESO. Thus, for managerial purposes, to understand the incentives and effects of such instruments becomes interesting.

As we mention in Chapter 2, according to Ingersoll (2006), there are three possible valuations when pricing ESOs. The first one is the risk-neutral valuation, that is, the value of the ESO if an unconstrained agent held it. Secondly, the subjective value that is made by a restricted ESO holder (the executive). Since the executive cannot trade with the ESO and his portfolio is not well diversified, his subjective value of the ESO should be lower than the risk-neutral valuation. Moreover, ESO adds more specific firm risk to the executive's portfolio, thus, ESO is exercised earlier (from the risk-neutral agent point of view). In consequence, the lack of diversification and the earlier exercise suppose an executive's ESO discount compared with the risk-neutral value. Finally, the objective value is the cost to the firm (fair value) of issuing the ESO. The objective value is made by a risk-neutral agent (the firm) but taking into account that the ESO's exercise is made by a restricted agent (the executive). The subjective value is lower than the risk-neutral valuation, and the objective one lies between both.

Motivated by the development of new accounting standards the problem of the ESO objective valuation has been studied in the last years[1]. Nevertheless, in this chapter we focus on the subjective ESO valuation and we adopt the certainty-equivalence (CE hereafter) principle. This framework identifies the executive's ESO value as the immediate ammount of cash that provides the same expected utility as the ESO does. In this case, the voluntary early exercise of the ESO is determined endogenously, and not imposed like in Chapter 2. We are interested in the voluntary early exercise behaviour, then we assume that the executive remains in the firm since ESO's maturity. This assumption implies no departure risk. Previous researches like Lambert et al. (1991), Huddart (1994), Kulatilaka and Marcus (1994), Hall and Murphy (2002), Tian (2004) and Cai and Vijh (2005), also employ the CE principle with the assumption of no departure risk. We will explain briefly these works in the next section remarking their main contributions and restrictions.

The main contribution of this chapter is twofold. First, we show how to combine the famous Least-Squares Monte Carlo technique for American options of Longstaff and Schwartz (2001) with the certainty-equivalence framework. The use of simulations in this framework is useful since it is very flexible and it allows us for considering American-style ESOs. Specifically, in this chapter we

[1]See Section 2.1 for a detailed discussion.

adapt to simulations the model of Hall and Murpy (2002) which considers an economy with a risk-free asset and the firm stock (one state-variable)[2]. Second, we show how to adapt easily the model to introduce time varying volatility using a two state Markov regime-switching, in contrast with the previous literature that only studies the case of constant volatility (see Section 3.2). Specifically, we analyze differences in the subjective valuation of the ESO and the expected exercise time for different levels of regime persistence, risk-aversion and diversification degree.

The rest of the chapter is organized as follows. In Section 3.2 we review the related literature. Section 3.3 describes our executive and the simulation-based algorithm. Some numerical results for the normal case are showed in Section 3.4. Section 3.5 conducts a numerical study under regime switching volatility. Finally, Section 3.6 concludes.

3.2 Previous Literature

During past decades, we have seen the growth of the use of compensation packages linked to the firms share price in order to provide incentives to the executive and reduce the agency problems (Jensen and Meckling, 1976). We briefly review some works that also use the certainty-equivalence to analyze the problem of the ESO subjective valuation.

[2]The more general case of two state-variables, introducing the market portfolio, is tackled in the next chapter.

3.2.1 Lambert, Larcker and Verrecchia (1991)

Lambert et al. (1991) analyze the valuation of compensation contracts from the executive's perspective. They were the first ones who identify the value of the compensation contracts as the amount of cash such that the executive is indifferent between receiving this payment for certainty or receiving the uncertain payoff that will be provided by the contract.

Specifically, they consider an executive who is offered a contract that at the end of the period will pay $Z(P)$ as function of the performance measure P[3]. Moreover, the remaining of executive's wealth is splitted into two components, one directly proportional to the performance measure P (MP) and the other is unrelated, $\bar{W}$.

The value of the executive's contract Z is the nonstochastic amount of cash CE, that leaves the executive indifferent between this amount and the uncertain payoff $Z(P)$, in other words, the amount CE that provides the same expected utility to the executive than the contract. Then, the value of the contract CE must satisfy the following equation

$$\int_0^\infty U(\bar{W} + MP + Z(P))f(P)dP = \int_0^\infty U(\bar{W} + MP + CE)f(P)dP$$

that is

$$\mathbb{E}\left[U\left(W_{Z(P)}\right)\right] = \mathbb{E}\left[U\left(W_{CE}\right)\right] \tag{3.1}$$

where, U represents the executive's utility function, with $U' > 0$, $U'' < 0$, that

[3]Note that depending on the form of $Z(P)$ and the definition of P, the contract can represent an annual bonus plan, ESO, a performance plan, etc...

is, the executive is risk-averse.

The main findings of the paper are:

1. CE is a weakly increasing function of $\bar{W}$. An increase in $\bar{W}$ implies that a larger proportion of executive's wealth is not linked to the performance measure P, thus the executive's contract discount (objective vale - subjective value) becomes smaller.

2. Assuming that the expected value of the executive's other wealth, $\bar{W} + MP$, remains constant, CE is weakly decreasing in M. The intuition of this finding is the opposite of the previous: as larger is M, more linked is the executive's wealth to P and more firm specific risk is supported. Thus, the contract discount becomes larger.

3. CE is a weakly decreasing function of his risk-aversion.

The results of this article are the cornerstone of the later models about executive compensation based on the certainty-equivalence principle. However, this initial setting has some limitations. First, they only consider the one-period case, which may be appropiate for European-style contracts but not for the American-style ones. Secondly, they restrict the non-option wealth to be invested in the risk-free asset ($\bar{W}$) or linked to the perfomance measure (MP), without the possibility to invest in the market portfolio according with the CAPM model. Finally, they don't provide a method to compute the objective (fair) value, or the firm cost of the performance contract $Z(P)$.

3.2.2 Kulatilaka and Marcus (1994)

Going forward to FASB 123 (1995), Kulatilaka and Marcus (1994) propose a method to account for the cost of executive stock options. Their proposal is motivated by the FASB draft (1993), which requires that ESOs should be recognized as compensation costs at the time these options are granted. The authors identify the early exercise as the main difference between tradable options and ESOs.

Kulatilaka and Marcus propose a model based on the binomial lattice of Cox et al. (1979). They assume that the executive has an initial non-stochastic wealth, W_0, that is invested in the risk-free asset. Moreover, he is granted with a package of ESOs with size N_{eso}. He determines his exercise rule maximizing the expected utility of his terminal wealth. They also assume that profits from the executive's ESO exercise are invested in the risk-free asset until maturity. Thus, conditional on ESO exercise at time t, the terminal wealth of the executive would be

$$W_{T|t} = W_0 e^{r_f T} + N_{eso} \left(S_t - K\right)^+ e^{r_f (T-t)} \tag{3.2}$$

where S_t is the stock price at time t, K is the exercise price and r_f is the risk-free rate of return.

They propose a two-step procedure to obtain the ESO fair value. At the first step, they build the binomial tree for the firm share price under the real/physical measure, and working backwards they obtain the exercise rule comparing the expected utility of exercise with the expected utility of holding the ESO one period ahead. Once the exercise nodes are obtained, the second step consists in building the binomial tree under the risk-neutral measure to calculate the option price under this measure, but with the exercise rule obtained under the real measure.

They find that the ESO value depends on both the executive's risk-aversion and the weight of W_0 over the total wealth. As expected, the higher the risk-aversion is, the lower the ESO subjective value. Furthermore, the lower the specific firm risk supported (higher W_0) by the executive, the larger the ESO value becomes. They also obtain an interesting counterintuitive behaviour: the ESO objective value can decrease as volatility increases. Since the executive is risk-averse, an increase in the volatility leads to an earlier exercise, which reduces the value of the option. This effect usually dominates the positive effects of volatility in tradable options. Finally, in contrast with tradable options, where the expected rate of return of the underlying asset is irrelevant, they also show that the ESO value is positively related to the expected underlying stock return.

With this procedure, one can approximate the firm's ESO cost. However, Kulatilaka and Marcus do not compute the executive's value through the certainty-equivalence like in Lambert et al. (1991). Moreover, they restrict the executive's investment possibilities to the risk-free asset, forgetting the possibility to invest in the market portfolio. They do not consider the lack of diversification due to the existence of other compensation instruments linked to the firm share price like restricted stock either.

At the same year, Huddart (1994) also proposes to use the binomial tree of

Cox et al. (1979) to value employee stock options. He also supposes that the ESO proceeds are invested in the risk-free asset instead of a mix with the market portfolio. However, he assumes an economy without systematic risk, that is, the market portfolio and the firm stock rate of returns equal the risk-free rate, which supposes a serious assumption compared with the model of Kulatilaka and Marcus (1994). Moreover, he does not consider the existence of other wealth related with the firm perfomance.

3.2.3 Hall and Murhpy (2002)

They employ the certainty-equivalence framework developed by Lambert et al. (1991) to analyze widely the value and cost of non-tradable American-style call options held by undiversified and risk-averse executives. They use an extension of the Kulatilaka and Marcus (1994) model. Specifically, besides N_{eso} non-tradable American call options, Hall and Murphy (2002) consider that the executive has a large proportion, α, of his non option wealth ,W_0, in restricted stocks of the firm, that cannot be sold until ESO's maturity. This assumption improves the model with respect to the one of Kulatilaka and Marcus (1994), since it accounts for the lack of diversification (extra specific risk) of the executive's wealth. Moreover, note that the proposal of Hall and Murphy (2002) nests the one of Kulatilaka and Marcus (1994) and they become the same for $\alpha = 0$. Hall and Murphy (2002) constrain the executive's investment opportunities set like in Kulatilaka and Marcus (1994), that is, the non stochastic wealth, $(1 - \alpha)W_0$, and the payoffs from the ESO exercise can only be invested in the risk-free asset. In this case, conditional on ESO exercise at time t, the terminal wealth of the executive would be

$$W_{T|t} = (1-\alpha)W_0 e^{r_f T} + \alpha W_0 \frac{S_T}{S_0} + N_{eso}\,(S_t - K)^+\, e^{r_f (T-t)} \tag{3.3}$$

Similar to Kulatilaka and Marcus (1994), they build a binomial tree under the real/physical measure to obtain the expected utility, the certainty-equivalence at the starting node and the exercise price thresholds. According with the findings from previous research they find that the executive's ESO value depends on the executive's risk-aversion and the size of the restricted stock over the whole of the non option wealth (diversification degree). Moreover, they obtain the objective ESO value simulating stock price paths under the risk-neutral measure and assuming the exercise price thresholds obtained under the real/physical measure. Respect to the objective ESO value they report that it is lower than the risk-neutral price (i.e., the Black-Scholes price for European options) and the ratio of the subjective (executive) ESO value over the objective (firm) one decreases as more risk-averse or less diversified (more restricted stock) the executive becomes. Thus, depending on these characteristics the ESO incentives also vary.

3.2.4 Tian (2004)

Using the certainty-equivalence framework of Lambert et. al (1991), Tian (2004) analyzes the incentive effects of ESO grants. Like in previous research, he considers a risk-averse executive with a power utility function. In this model the executive is granted with N_{eso} European-style ESOs, and he optimally invests his non restricted wealth between the market portfolio and the risk-free asset.

Then, the executive's terminal wealth is

$$W_T = W_0 \left[\eta \frac{M_T}{M_0} + (1-\eta) e^{r_f T} \right] + N_{eso} \left(S_T - K \right)^+ \tag{3.4}$$

where M_T and M_0 are the market portfolio price at maturity and grant date respectively and η is the proportion of non restricted wealth invested in the market portfolio. The main findings of the article are as follows. First, the incentive to increase stock price does not increase monotonically as proportional option wealth increases. Second, if the proportional option wealth exceeds a critical threshold, granting more ESOs reduces incentive strength and becomes counterproductive. Third, ESO creates incentives to reduce (increase) idiosyncratic (systematic) risk. Finally, the ESO moneyness at the grant date has a substantial impact on incentive effects.

From equation (3.4), note that this setting does not include restricted stock in the executive's wealth. Moreover, he does not allow ESO early exercise, that is, he only analyzes the less interesting case of European-style ESOs.

3.2.5 Cai and Vijh (2005)

They propose a numerical method in order to solve numerically the executive problem considering American-style ESOs and restricted stocks in a two state-variable framework. Their model considers an executive that holds a large proportion of his wealth in restricted stock of the company and he is granted with some ESOs as in Hall and Murphy (2002). However, in this case, the executive optimally splits his non restricted wealth between the market portfolio and the risk-free asset. They obtain that not considering the market portfolio in the executive's investment opportunity set, as in Huddart (1994), Kulatilaka and Marcus (1994) and Hall and Murphy (2002), may leads to some inconsistent ESO subjective valuation.

As the authors point out, for N periods, their model generates $(N+1)(N+2)(2N+3)/6$ nodes and they only can consider four periods per year for a ten-year option. Thus, the computational requirements of the method seems to be a serious drawback in order to implement some extensions of the model. Moreover, their numerical technique can not be extended to accomodate more than two state-variables.

3.3 The model

In this section we approach the subjective valuation of non-tradable stock options from a restricted executive point of view using simulations. As we discussed previously, our model consists on a simulation-based algorithm of the Hall and Murphy (2002) model (benchmark). The introduction of the market portfolio as in Tian (2004) and Cai and Vijh (2005) will be analyzed in Chapter 4. We first describe the executive's preferences, endowments and restrictions, and second we explain the simualtion-based algorithm.

3.3.1 The executive framework

The proposed valuation method is based on the certainty-equivalence principle. That is, it identifies the subjective ESO value with the amount of cash delivered at the grant date that invested until ESO maturity reports the same expected utility to the executive.

We assume that the executive has an initial wealth W_0 of which a proportion α comprises restricted stocks of the company that can not be sold until ESO maturity. The larger α is, the more related the executive's wealth with the firm stock price is, and the higher specific firm risk is supported. Thus, a higher value of α represents a less-diversified executive. The remainder of the non ESO executive's wealth, $(1-\alpha)W_0$, is invested in the risk-free asset. Moreover, the executive is granted with N_{eso} non-transferable American-style call options on the company stock with an exercise price of K. Thus, conditional on non previous option exercise, the executive's wealth at maturity (T) is

$$W_{T|T} = W_0 \left[\alpha \frac{S_T}{S_0} + (1-\alpha)e^{r_f T} \right] + N_{eso}\left(S_T - K\right)^+ \tag{3.5}$$

where $W_{i|j}$ indicates the executive's wealth at time i conditional on ESO exercise at time $j \leq i$, S_i is the stock price at time i and the expresion $(b)^+$ represents the maximum between b and zero. From previous equation, note that for comparative reasons with Hall and Murpy (2002) we are assuming that the firm stock does not pay dividends[4].

[4] In the next chapter we conduct a sensitivity analysis showing the effects of the dividend yield in the more general setting of two state-variable framework.

If the executive decides to exercise the ESOs at any time t prior to the maturity, we assume he invests the ESO payoffs in the risk-free asset, which is mantained until T. Then, his terminal wealth in the case of exercise at time t is

$$W_{T|t} = W_0 \left[\alpha \frac{S_T}{S_0} + (1-\alpha) e^{r_f T} \right] + N_{eso} max\left(S_t - K, 0\right) e^{r(T-t)} \tag{3.6}$$

Note that despite the ESO early exercise, the executive must maintain his restricted stock until maturity.

We suppose that firm the stock price evolves according the next equation

$$S_{t+1} = S_t \exp\left(\mu_s - \frac{1}{2}\sigma_s^2 + \sigma_s z_{t+1}\right) \tag{3.7}$$

where $\mu_s = log\left(1 + r_f + \beta\left(r_m - r_f\right)\right)$ is the expected rate of return of the firm's stock, σ_s is the corresponding volatility, β is the systematic risk of the stock, i.e. $\beta = \rho\frac{\sigma_s}{\sigma_m}$ and z follows the standard normal distribution.

We consider a risk-averse executive with a power utility function with respect to his wealth. The main properties of this utility function are $U'(W) > 0$, the executive prefers more to less, and $U''(W) < 0$, the executive is risk-averse and may be expressed as

$$U(W) = \begin{cases} \frac{W^{1-\gamma}}{1-\gamma}, & \text{if } \gamma \neq 1 \\ \\ log(W), & \text{otherwise,} \end{cases} \tag{3.8}$$

where $\gamma > 0$ is the relative risk-aversion coefficient: the higher γ, more risk-averse the executive becomes. This utility function also has been used previously by Kulatilaka and Marcus (1994), Hall and Murphy (2002), Tian (2004) and Cai and Vijh (2005), among others.

The executive maximizes the expected utility of his terminal wealth. Therefore, the executive's problem may be expressed as

$$\max_{t} \ \mathbb{E}_0 \left[U \left(W_{T|t} \right) \right]$$

As we mentioned above, solving the previous optimization program leads to obtain the optimal ESO exercise policy. This fact complicates the problem and some previous articles avoid this trouble considering only the case of European style ESO ($t^* = T$), like in Johnson and Tian (2000) for indexed executive stock options and Tian (2004).

We calculate the ESO certainty-equivalence to obtain the subjective ESO valuation, that is, we find the amount of cash CE that provides the same expected utility than holding the ESO. Then, the total executive's wealth at maturity when an amount of cash equal to CE is delivered in place of the ESOs at grant date is

$$W_T^{CE} = W_0 \left[\alpha \frac{S_T}{S_0} + (1-\alpha) e^{r_f T} \right] + N_{eso} CE \qquad (3.9)$$

Thus, to obtain the subjective value of the ESO we solve for CE the following expression according to equations (3.5) and (3.9)

$$\mathbb{E}_0 \left[U \left(W_T^{CE} \right) \right] = \mathbb{E}_0 \left[U \left(W_{T|t^*} \right) \right] \qquad (3.10)$$

where the subindex in the expectation operator indicates the grant date.

3.3.2 The utility-pricing algorithm

In this section we describe our algorithm to obtain the ESO subjective value. If the two expectations from equation (3.10) were known with certainty, it would be easy to find numerically CE by a quadratic distance minimization like this[5]

$$\min_{CE} \left(1 - \frac{\mathbb{E}_0 \left[U \left(W_T^{CE} \right) \right]}{\mathbb{E}_0 \left[U \left(W_{T|t^*} \right) \right]} \right)^2 . \tag{3.11}$$

Computing the left-hand side of equation (3.10) is straightforward for given simulated samples of the stock price S_t according to equation (3.9). However, obtaining $\mathbb{E}_0 \left[U \left(W_{T|t^*} \right) \right]$ is the main drawback to solve equation (3.11), since this expected utility involves the optimal exercise of the ESO.

We propose to adapt the least-squares Monte Carlo (LSMC) approach. In short, let X be the state-variable and $Y(X)$ a function that depends on this state-variable, then, the LSMC method computes the conditional expectations of Y, that is $\mathbb{E}[Y|X]$, by least-squares over some basis function of a simulated sample of X. In the risk-neutral option valuation case, X is the current stock price and Y is the one period ahead discounted value of the derivative. In our utility framework, X also comprises the current stock price but Y is the utility of the executive's wealth one period ahead over the terminal wealth.

[5] We use a relative quadratic distance instead of the absolute one in order to avoid numerical problems in the minimization. Note that depending on the risk-aversion parameter the expected utility is of order 10^{-7} or 10^{-13}.

Now, we present the algorithm that allows us to obtain the expected utility at $t = 0$ according to the executive's features in Section 4.2.1. The main difference between the risk-neutral case and our utility setting is that in the former the agent determines the exercise rule comparing the certain payoff of immediate exercise with the expectation of the discounted future value of the option. Meanwhile, our restricted agent compares the current expected utility of his terminal wealth if he exercises the ESO package with the current expected utility of his terminal wealth in case of holding.

We start simulating m price paths under the real/physical measure according to equation (3.7). Let $\mathcal{U}$ be a vector of size $(m \times 1)$ where we shall collect the optimal utilities obtained from the executive's hold/exercise decision for every path. At each time step where the ESO is exercised, this vector will be updated with the maximizing utilities. At maturity, the executive exercises the ESO if they are in the money. In consequence, the utility is computed using the utility function on the terminal wealth following equation (3.5) for all simulated paths. Therefore, if exercise is only allowed at maturity the vector of maximizing utilities, $\mathcal{U}$, coincides with $U\left(W_{T|T}\right)$.

One period before, at $T-1$, the executive has to decide between holding or exercising the ESO for all in the money paths. He will exercise (hold) the ESO when the expected utility of exercise becomes larger (smaller) than the expected utility to continue with the ESO for one period, that is, for those paths in the money at $T-1$, the ESO is exercised when

$$\mathbb{E}_{T-1}\left[U\left(W_{T|T-1}\right)|S_{T-1}\right] > \mathbb{E}_{T-1}\left[\mathcal{U}|S_{T-1}\right]. \tag{3.12}$$

To obtain $\mathbb{E}_{T-1}\left[U\left(W_{T|T-1}\right)|S_{T-1}\right]$ we first calculate $U\left(W_{T|T-1}\right)$ using equation (3.6) and the utility function. Second, we estimate the time $T-1$ expectation regressing $U\left(W_{T|T-1}\right)$ over some basis functions of the current state-variable, S_{T-1}. The projected value of the previous regression is the conditional expected utility, that is,

$$\mathbb{E}_{T-1}\left[U\left(W_{T|T-1}\right)|S_{T-1}\right] = \widehat{U}\left(W_{T|T-1}\right) \tag{3.13}$$

The expected utility to continue one period with the ESO, $\mathbb{E}_{T-1}\left[\mathcal{U}|S_{T-1}\right]$, is also computed by least-squares. In this case we regress the one period ahead maximizing utility, $\mathcal{U}$, over the same basis functions of the state-variable. The projected values of this regression are the expected utilities when the executive continues with the ESO

$$\mathbb{E}_{T-1}\left[\mathcal{U}|S_{T-1}\right] = \widehat{\mathcal{U}} \tag{3.14}$$

The exercise rule is obtained by comparison of equations (3.13) and (3.14) for each path in the money. For those paths where equation (3.12) is verified, that is, the ESO is exercised, $\mathcal{U}$ is updated with the corresponding expected utility $\widehat{U}\left(W_{T|T-1}\right)$. We can work backwards with this scheme. In general, at any exercise point t we have to compute $\mathbb{E}_t\left[U\left(W_{T|t}\right)|S_t\right]$ and $\mathbb{E}_t\left[\mathcal{U}|S_t\right]$ by least-squares. Once $\mathcal{U}$ has been updated until $t=0$ we solve numerically for CE in equation (3.11).

3.4 Numerical Study

In this section we verify numerically the validity of the previous algorithm (LSMCU henceforth) comparing its results with the ones from the article of Hall and Murphy (2002) obtained using a binomial lattice (HM henceforth). First we focus on the subjective valuation, secondly we compare the objective ESO values and finally we conduct a sensitivity analysis about the effects of the vesting period.

Along the paper we use a general parameter setting although we alter some of them in order to reveal some interesting results about the subjective valuation. The executive has 5 million dollars in initial wealth comprising company stock and safe cash. Moreover, the executive is granted with 5,000 ten-year stock options issued at the money (the usual way), with an exercise price of 30. We take $\sigma = 0.30$, $\beta = 1$, $r_f = 6\%$ and $r_m - r_f = 0.65$. For the risk-aversion (γ) and the degree of diversification (α) we consider we consider four different combinations showed in Table 3.1. In general, our parameters are identical to those used by HM for comparison.

Every result obtained with the LSMCU method is the average of 100 previous estimations obtained with 50 different seeds plus the 50 antithetics, and each estimation has been obtained running 200,000 price paths with twelve time steps per year (monthly frequency)[6]. That is, we approximate the ESO subjective value with the value corresponding to a Bermudan-style option with 120 exercise dates. About the basis functions, when using powers of S_t, M_t and their cross

[6]We have observed that the model produces more accurate prices when enlarging the number of simulated paths instead of when we increase the frequency of simulations (weekly or daily).

Table 3.1: Risk-aversion and diversification degrees.

	α	γ
case 1	1/2	2
case 2	1/2	3
case 3	2/3	2
case 4	2/3	3

This table shows the four different combinations of executive's risk-aversion (γ) and diversification degree (α) considered across this chapter.

product, usually the inverse of the matrix of basis functions is not well behavied. We detect that taking powers of $\log(S_t)$, $\log(M_t)$ and their cross product, we avoid this problem.

3.4.1 The subjective value

In Table 3.2 we compare the subjective value obtained with the LSMCU method with the ones from the model of HM, and all columns labelled with the HM subindex reflect data copied from their article. The first column shows some couples of the diversification and risk-aversion degree parameters (α and γ respectively). The BS column displays the Black-Scholes (1973) price of our hypothetical ESO, which does not depend on the particular preferences and wealth of the executive. The third and fourth columns represent the percentage over BS price of the ESO subjective value for the European-style ESO. We can observe that the executive's discount for the European ESOs are quite similar for the two methods, i.e., for $(\alpha, \gamma) = (1/2, 2)$ the executive's discounts are 63.5%

and 62.38% respectively. For the American ESO case (fifth and sixth columns) we display the absolute price, in contrast with E columns. We also observe quite similar prices although we detect a small systematic overvaluation of the LSMCU method for all (α, γ) pairs considered. Below all prices based on simulations we display, in parenthesis, the price standard deviation calculated over the 100 estimations. As expected, the general result is that the subjective ESO value is negatively related with diversification degree α and the executive's risk-aversion γ.

Finally, the last two columns reflect the subjective *delta* of the American-style ESO at grant date, that is, the partial derivative of the subjective ESO value with respect to the stock price, $\partial A_{[...]}/\partial P$. The *delta* is interesting in the executive's compensation problem since it is an indicator of the possible incentive effect of the ESO. We may observe the same behaviour in Δ_{sim} than in Δ_{HM}: the incentive (measured by Δ) reduces (increases) when more (less) risk-averse (higher γ) and more (less) undiversified (higher α) is the executive. However, these values present some differences. There are two possible reasons for these differences. First, we compute Δ_{sim} by perturbating slightly the starting price P_0 and taking the first difference, $\Delta_{sim} = \left(ESO(P_0 + \varepsilon) - ESO(P_0 - \varepsilon)\right)/2\varepsilon$, and depending on the choice of ε the value of Δ_{sim} may vary. Secondly, in Hall and Murphy (2002) it is not clear how they compute Δ_{HM}. They could compute it for each node on the tree for all time steps and take the average as Δ_{HM} instead of to consider only the *delta* on the grant date.

Table 3.2: Binomial tree vs. Simulation-based subjective valuation

(α, γ)	BS	E_{HM}	E_{sim}	A_{HM}	A_{sim}	Δ_{HM}	Δ_{sim}
(1/2, 2)	16.708	63.5	62.38	12.40	12.767	0.61	0.678
			(0.031)		(0.242)		(0.071)
(1/2, 3)	16.708	36.7	36.57	9.42	9.802	0.53	0.602
			(0.021)		(0.217)		(0.071)
(2/3, 2)	16.708	44.8	43.67	9.96	10.396	0.56	0.650
			(0.025)		(0.266)		(0.066)
(2/3, 3)	16.708	21.1	20.549	7.33	7.624	0.49	0.587
			(0.015)		(0.278)		(0.060)

This table shows the subjective value of a European and American style ESO for different values of the risk-aversion and diversification degree (α, γ). The ESO has a maturity of 10 years, $r = 0.06$, $\beta = 1$, $r_m - r_f = 0.065$, $\sigma = 0.3$ and the stock does not pay dividends. We assume $W_0 = 5,000,000$ split in safe cash and restricted stock according to parameter α and $N_{opt} = 5000$. We assume no vesting period. Simulations have been done with monthly frequency and each price is the average of 100 prices (50 plus the 50 antithetics) obtained with 200,000 paths. Below each price, in parenthesis, it is displayed the standard deviation. The prices have been obtained solving for the certainty-equivalence.

3.4.2 The objective valuation

As Ingersoll (2006) points out, the objective value of the ESO is the cost to the firm and this value recognizes the sub-optimal exercise but from a risk-neutral agent (the firm) perspective. Thus, the objective value may be approximated considering the risk-neutral price process but with the exercise decision made by the (undiversified) executive.

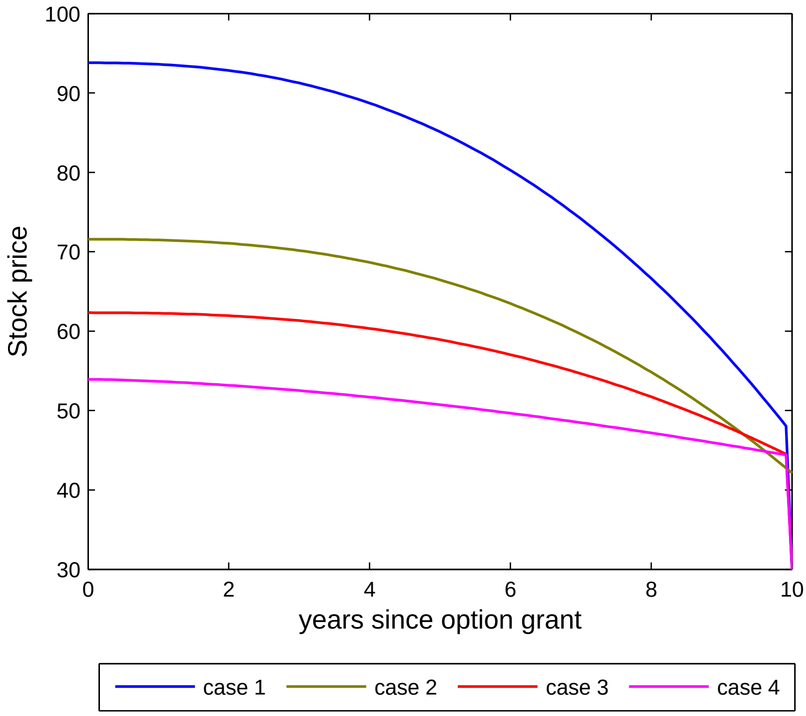
In Figure 3.1 we show the threshold exercise prices implied on LSMCU results from Table 3.2. Each line corresponds to a different (α, γ) pair. The price threshold reflects a time-varying frontier where the executive is indiferent between to hold or to exercise the ESO. Above the threshold the executive will choose to exercise and below the threshold the opposite. These thresholds have been calculated as follows: first we take the minimum stock price at which the ESO is exercised for each time step over the $200,000$ simulated paths, and second we take the average of the minimums over the 100 simulations (50 plus the 50 antithetics)[7]. The first result is that the more risk-averse and/or less-diversified is the executive, the lower the price threshold. Note that a risk-neutral agent never would exercise the ESO in this case since we are not considering dividend payments.

In Table 3.3 we display the objective values using the price thresholds from Figure 3.1 and those from Hall and Murphy (2002). Columns HM and LSMCU are very similar, revealing that the LSMCU and the binomial tree method produce very similar objective values. The columns labelled as 'Disc.' have been computed as the ratio $ESO_{sub}/ESO_{obj} \times 100$ and they reflect the executive's discount over the ESO's firm cost, i.e. for $(\alpha, \gamma) = (2/3, 3)$ the executive's subjective valuation is around the 64% of the firm ESO cost[8]. Observe that the executive's discount depends on the diversification degree and his risk-aversion; for $(\alpha, \gamma) = (2/3, 3)$ the subjective value represents the 64% of the objective one, while for $(\alpha, \gamma) = (1/2, 2)$ the discount ratio is 85%.

[7]The price thresholds showed in Figure 3.1 have been smoothed using a power function, $(a + bx^c)$.

[8]Note that a higher discount ratio supposes lower differences between the subjective ESO value and the objective one.

Figure 3.1: Executive price thresholds



This figure shows the executive's price thresholds, that is, the stock price where the executive is indifferent between to hold or to exercise the ESO. Each line correspond with a different level of risk-aversion and diversification degree according to Table 3.1. We have used a power function to smooth the threshold prices. The remainder of the parameters are described in Section 3.4.

Table 3.3: Binomial tree vs. simulation-based objective valuation

$(\alpha,\ \gamma)$	BS	HM	Disc.$_{HM}$	LSMCU	Disc.$_{sim}$
(1/2, 2)	16.708	14.76	84.0 %	14.922 (0.056)	85.6 %
(1/2, 3)	16.708	13.06	72.1 %	13.055 (0.041)	75.1 %
(2/3, 2)	16.708	13.60	73.2 %	13.650 (0.044)	76.2 %
(2/3, 3)	16.708	11.57	63.4 %	11.766 (0.031)	64.8 %

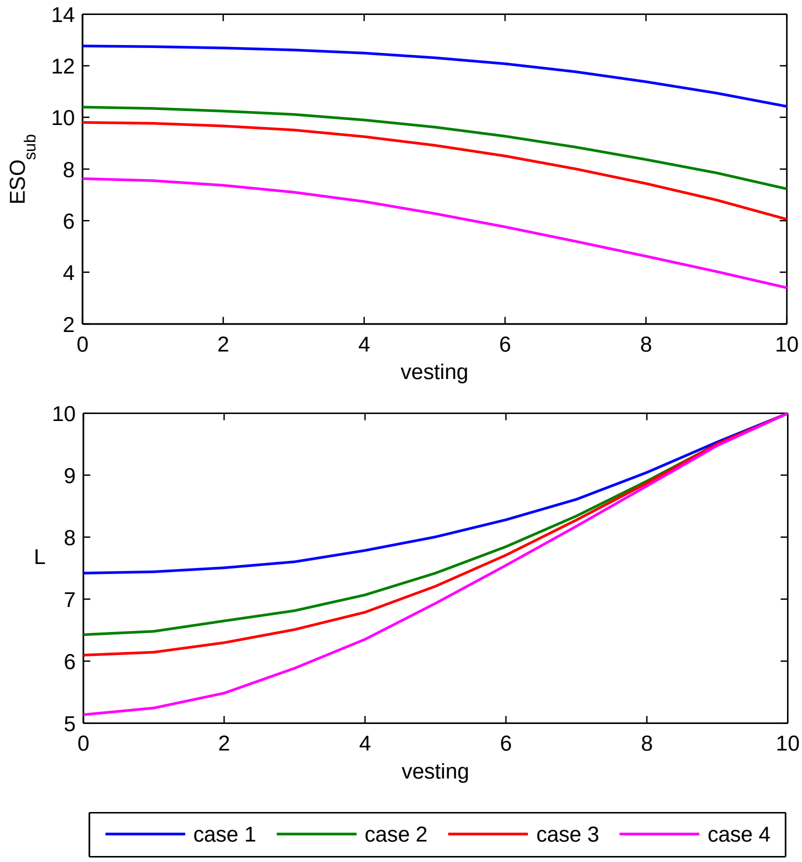
This table shows the objective value of an American style ESO for different values of the risk-aversion and diversification degree (α, γ). The ESO has a maturity of 10 years, $r = 0.06$, $\beta = 1$, $r_m - r_f = 0.065$, $\sigma = 0.3$ and the stock does not pay dividends. We assume $W_0 = 5,000,000$ split in safe cash and restricted stock according to parameter α and $N_{opt} = 5000$. We assume no vesting period. Simulations have been done with monthly frequency and each price is the average of 100 prices (50 plus the 50 antithetics) obtained with 200,000 paths. Below each price, in parenthesis, it is displayed the price standard deviation. The prices have been obtained valuing an American option under the risk-neutral measure but with the exercise rule determined by the executive's price threshold.

3.4.3 The vesting period

We extend our analysis and now we focus on an important feature of the ESOs, the vesting period. In Figure 3.2 we display in the upper graphic the subjective ESO price depending on the vesting period length for the four (α, γ) pairs considered, whereas in the bottom graphic we show the expected exercise time, denoted by L. L is obtained by averaging the exercise time for each simulated path and for the 100 estimations (50 plus the 50 antithetics). Note that for $\nu = 0$ we have a fully American ESO meanwhile for $\nu = 10$ we have the European one. As expected, the ESO is more valued by the executive as shorter is the vesting period. Note that the expected exercise time, L, enlarges with the vesting period, as expected. Moreover, we can see that the larger the risk-aversion (γ) and the restricted stock weight in total wealth (α), the shorter the expected life of the ESO.

The upper graphic in Figure 3.3 represents the objective ESO values for different vesting period lengths using the price thresholds from Figure 3.1. The main conclusion is that the objective ESO value (the firm cost) increases with the vesting period length. We would like to remark that this result is the opposite for the subjective ESO value, that is, the vesting period affects in an opposite way both the objective and subjective ESO values. On one hand, as shorter is the vesting period, there are more exercise opportunities, thus the ESO provides more utility to the executive increasing his (subjective) valuation. On the other hand, a larger vesting period avoids a more earlier (and more suboptimal) exercise, and it produces an increase in the ESO cost, despite the fall in the executive's utility. This fact is known as the vesting protection phenomenon and also has been

Figure 3.2: ESO subjective value and vesting period



This figure represents the subjective ESO value (upper graphic) and the expected exercise time (bottom graphic) for different vesting period lengths ranging from 0 to 100% of the time to maturity of the ESO. Each line corresponds to different values for the executive's risk-aversion and diversification degree according to Table 3.1. The remainder of the parameters are described in Section 3.4.

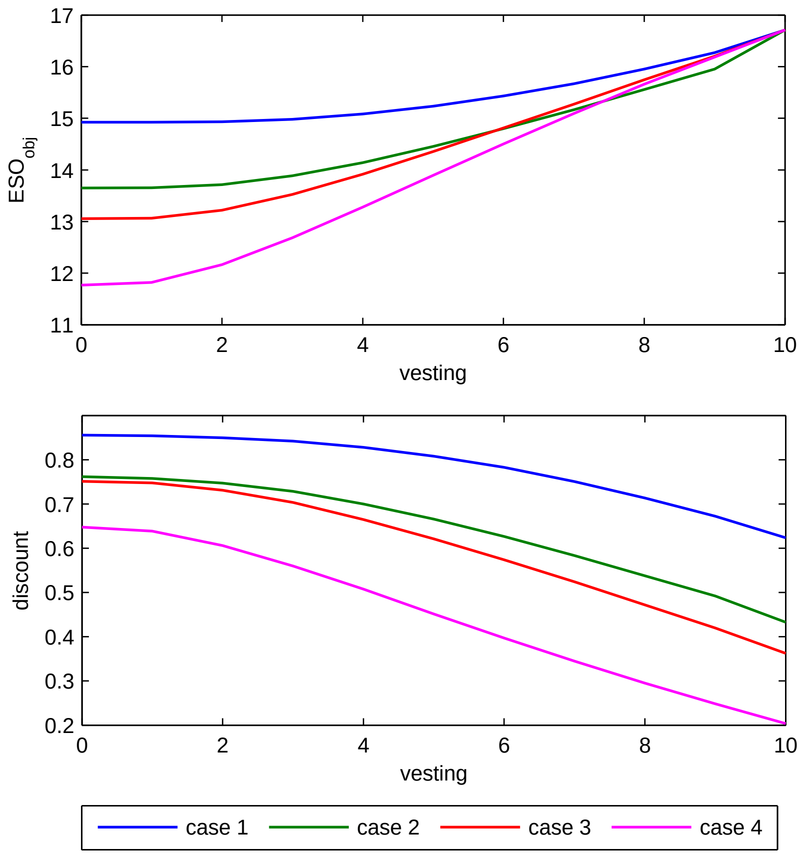
observed by Kulatilaka and Marcus (1994) in a simpler model, among others.

In the bottom graphic of Figure 3.3 we represent the ratio of the subjective values over the objective ones. According with the previous results, this ratio decreases with the vesting period length. This ratio is relevant for compensation packages design since it indicates the difference between both values, the firm ESO cost and the executive's incentive. Summarizing, ESOs with shorter vesting periods are more attractive for executives and the associated firm costs are lower compared with a larger vesting period ESO.

3.5 Subjective ESO valuation under switching volatility

In this section we analyze the effects of time varying volatility in the subjective ESO valuation, since empirical evidence shows that the assumption of constant volatility is almost constantly violated. Moreover, abrupt changes are a prevalent feature of financial data. Specifically, we consider now that the stock return is drawn from a mixture of two normal densities. We assume the economy switches between two states. The state in each period determines the normal distribution that generates the return for that period. The state is assumed to be generated by a first-order Markov process. The states are characterized by the variances of their densities as low-variance state and high-variance state. Switching-regime models were introduced by Hamilton (1989) to analyze time series with shifts. Turner et al. (1989) also use a switching regime model to

Figure 3.3: Objective ESO value and vesting period



This figure represents the objective ESO value (upper graphic) and the executive's discount (bottom graphic), calculated as ESO_{sub}/ESO_{obj}, where ESO_{sub} is the subjective ESO valuation from Figure 3.2, for different vesting period lengths ranging from 0 to 100% of the time to maturity of the ESO. Each line corresponds to different values for the executive's risk-aversion and diversification degree according to Table 3.1. The remainder of the parameters are described in Section 3.4.

account for the risk-return relation (leverage effect). Risk-neutral option pricing under switching regimes have been analyzed in Naik (1993) and Bollen (1998), among others.

3.5.1 Model setup

Let $y_t = log(S_t/S_{t-1})$ the logarithmic stock return. We assume it follows a Markov Regime-Switching (MRS) process

$$y_t = \mu_s - \frac{1}{2}\sigma_t^2 + \sigma_t z_t \tag{3.15}$$

where z_t is a standard normal innovation and σ_t^2 is

$$\sigma_t^2 = \sigma_0^2(1 - \Psi_t) + \sigma_1^2(\Psi_t) \tag{3.16}$$

and Ψ_t denotes the state (regime) that the stock lies in at time t. We assume that Ψ_t follows a Markow chain with two regimes driving the stock volatility, i.e. $\Psi_t \in \{0, 1\}$. The movements between the two regimes are dictated by the transition matrix of probabilities, **P**, given by

$$\mathbf{P} = \begin{bmatrix} p_{00} & p_{10} = 1 - p_{11} \\ p_{01} = 1 - p_{11} & p_{11} \end{bmatrix}$$

where p_{ij} is the transition (switching) probability of moving from state i to state j and p_{ii} is the persistence of regime i. The first regime denoted by "0", is characterized by the variance σ_0^2, while the second regime by σ_1^2. The unconditional probability that the process will be either in regime 0 or 1 at any given date is

$$
\begin{aligned}
P\{\Psi_t = 0\} &= \frac{1 - p_{11}}{2 - p_{00} - p_{11}} \\
P\{\Psi_t = 1\} &= \frac{1 - p_{00}}{2 - p_{00} - p_{11}}
\end{aligned} \tag{3.17}
$$

These probabilities will be denoted as π_0 and π_1 respectively.

We assume that the processes $\{\sigma_t^2\}$ and $\{z_t\}$ are stochastically independent. The unconditional variance is

$$\mathbb{E}\left[\sigma_t^2\right] = \pi_0\sigma_0^2 + \pi_1\sigma_1^2 \tag{3.18}$$

and their unconditional density is a mixture of normal densities,

$$f(y_t) = \pi_0\ \phi(y_t|\mu, \sigma_0^2) + \pi_1\ \phi(y_t|\mu, \sigma_1^2) \tag{3.19}$$

where $\phi(y_t|\mu, \sigma_i^2)$ represents the density of $N(\mu, \sigma_i^2)$.

3.5.2 Numerical results

To implement the numerical exercise, we establish $\sigma_0 = 0.25$ and $\sigma_1 = 0.45$. Moreover, we suppose that the low-volatility regime is very persistent, $p_{00} = 0.99$, and we propose different values for the high-volatility regime persistence (p_{11}). We also suppose that volatility shifts are unobserved. To simulate the MRS process we determine the initial state using the unconditional probabilities in equation (3.17). We generate a vector u of uniform random numbers between

0 and 1, and we establish the initial state as "0" ("1") for those paths where $U < \pi_0$ ($U > \pi_0$).

In Table 3.4 we show the percentage relative bias of either the subjective ESO valuation (Panel A) and the expected exercise time (Panel B) when considering the MRS model with respect to the constant volatility case. The biases have been computed as $bias = \frac{ESO_{sw} - ESO_{cv}}{ESO_{cv}} \times 100$, where ESO_{sw} is the price under switching volatility and ESO_{cv} is the one under the corresponding constant volatility. Valuation under constant volatility has been obtained using the square root of the unconditional variance according to equation (3.18) showed in column $\sqrt{\mathbb{E}\left[\sigma_t^2\right]}$. Column π_0 diplays the unconditional probability to lie in the low-variance state according to equation (3.17). The last four columns show the biases for the different cases of risk-aversion and diversification degree in Table 3.1.

We can appreciate a significant negative bias when introducing two switching volatility regimes. The negative bias supposes an overvaluation when considering the unconditional volatility in the constant volatility model[9]. We observe that bias reduces (in absolute value) when increasing the persistence of the high-volatility regime and for $p_{11} = p_{00}$ (last row), the bias is neglictible. Moreover, bias size diminishes stronger with the risk-aversion than with the diversification degree, i.e. for $p_{11} = 0.8$ the bias is around -16.4% for $(\alpha,\ \gamma) = (1/2,\ 2)$ and it drops to a -11.62% for $(\alpha,\ \gamma) = (1/2,\ 3)$, meanwhile the bias only diminishes to -15.541% for $(\alpha,\ \gamma) = (2/3,\ 2)$. Therefore, the executive discounts stronger the ESO when the true data generating process is a Markov volatility regime-

[9]The negative bias is in accordance with the results under the GARCH framework in Chapter 2.

Table 3.4: ESO subjective bias and switching volatility

			$(\alpha,\ \gamma)$			
p_{11}	π_0	$\sqrt{\mathbb{E}\left[\sigma_t^2\right]}$	(1/2, 2)	(1/2, 3)	(2/3, 2)	(2/3, 3)
			Panel A: ESO bias			
0.8	0.952	0.263	-16.395	-11.625	-15.541	-9.683
0.9	0.909	0.274	-13.328	-9.098	-12.526	-7.898
0.95	0.833	0.293	-8.941	-5.903	-8.480	-4.534
0.99	0.5	0.364	-0.424	-0.799	-0.577	-0.481
			Panel B: Expected exercise time bias			
0.8	0.952	0.263	-18.058	-11.756	-16.943	-10.260
0.9	0.909	0.274	-15.086	-8.835	-13.192	-7.497
0.95	0.833	0.293	-10.624	-6.232	-9.529	-4.858
0.99	0.5	0.364	-0.258	-1.261	-0.956	-1.394

This table shows the percentage relative bias of either the ESO subjective valuation (Panel A) and the expected exercise time (Panel B)bwhen the volatility process follows a two-state Markov regime-switching with respect to the constant volatility case. The volatilities under the two regimes are $\sigma_0 = 0.25$ and $\sigma_1 = 0.45$ respectively. The biases have been calculated as $bias = \frac{ESO_{sw} - ESO_{cv}}{ESO_{cv}} \times 100$, where ESO_{sw} and ESO_{cv} are the price under switching volatility and under the corresponding constant volatility. The corresponding constant volatilities have been obtained using equation (3.18) for the unconditional variance, showed in the third column. The persistence of the state ”0” is $p_{00} = 0.99$ and we consider different values for p_{11} in the first column. The second column exhibits the unconditional probability of lie in state ”0”. The last four columns show the biases for different levels of risk-aversion and diversification degree. The remainder of the parameters are described in Section 4.3.1.

switching model instead of when the volatility of the data generating process is constant. In Panel B we observe that the expected exercise time shows the same behaviour than the ESO subjective value. The switching volatility process supposes a significant reduction in the expected exercise time of the ESO respect to the constant volatility case. Moreover, the lower the difference between the unconditional probabilities, the lower the bias becomes. The risk-aversion and the diversification degree also reduce the L bias.

3.6 Concluding Remarks

In this chapter we show how to combine simulations with the certainty-equivalence framework to solve the subjective ESO valuation problem. We adapt the algorithm of Longstaff and Schwartz (2001) to obtain the expected utility of the terminal wealth of a restricted and undiversified executive. Since our model is based on simulations, it is suitable to deal with multiple state-variables (stock price and volatility) and it allows early exercise (American-style ESO).

We verify numerically the simulation-based algorithm comparing the subjective ESO values, the objective ones and the subjective ESO delta's obtained using our algorithm with the results from previous articles. Moreover, we show how the ESO subjective value is negatively related with the executive's risk-aversion and his diversification degree. We also analyze the vesting period effect obtaining that the shorter the vesting period, the higher the subjective valuation.

About the objective ESO value or firm ESO cost, we use the exercise price thresholds. We find that the objective value is lower than the risk-neutral valuation for all cases considered suggesting the Black-Scholes overvaluation when accounting for ESO cost. Moreover, the objective value increases with the vesting period length, just the opposite of the subjective value does, showing the vesting protection phenomenon. In consequence, the executive discounts more ESO values for long-term vested ESO, reducing its incentive. We also verify that our algorithm produces very similar values for the executive's discount than the previous based on binomial lattices. We also obtain that, depending on the risk-aversion and diversification degree, the subjective ESO value may be the 60% of the objective value, suggesting a relevant executive's discount.

Finally, as an example of the flexibility of our algorithm, we introduce a time varying volatility using a Markov regime-switching model. We consider that there is a very persistent low-variance regime and a high-variance regime. We obtain that the switching volatility produces an undervaluation when the value is compared with the one obtained under constant volatility. The bias may be higher than 15% and the bias size negatively depends on the persistence of the high-variance regime, the risk-aversion and the diversification degree. We obtain similar results for the expected exercise time, suggesting an earlier exercise when the variance is time varying.

4

Subjective Executive Stock Option Valuation: the Two State-Variable Framework

4.1 Introduction

The analysis of the subjective valuation of executive stock options (ESO hereafter) began with the pioneer article of Lambert et al. (1991). In this paper they identify the subjective ESO value with the certainty-equivalence of the ESO, that is, the amount of cash at grant date that reports the same utility to the executive. As Ingersoll (2006) points out, the subjective value is obtained from the point of view of a risk-averse, undiversified and restricted executive, and this value should be lower than the risk-neutral valuation. The certainty-equivalence principle has been used in Huddart (1994), Kulatilaka and Marcus (1994), Hall and Murphy (2002) or in Tian (2004)[1], among others, to study either the subjective ESO value and the ESO incentives. Carpenter (1998) also uses a utility-based model to conduct an empirical research and Henderson (2005) analyzes the problem of the subjective valuation and incentives in a continuous-time framework using utilities.

All these works make some assumptions to simplify the procedure to obtain the ESO subjective value. On one hand, some of them only consider European-style contracts. This assumption facilitates the pricing algorithm since the exercise rule becomes obvious. However, the vast majority of the ESOs are of American-style. On the other hand, other papers consider American-style contracts but in the one state-variable (the firm stock price) framework, avoiding to include the market portfolio as a component in the executive's wealth according with portfolio theory. This assumption simplifies the problem because the executive has not to find the optimal portfolio for his outside wealth. In general,

[1] See Section 3.2 for a more detailed discussion about these articles.

these papers assume that the whole of the executive's outside wealth is invested in the risk-free asset.

Cai and Vijh (2005) adapt the rainbow grid of Rubinstein (1994) in order to consider American-style contracts and the two state-variable framework. They find that ESO subjective value under the one state-variable framework can be erroneous depending on the model parameters. In this chapter we adapt the LSMCU algorithm developed in Chapter 3 to include the market portfolio in the executive's investment opportunity set as in Cai and Vijh (2005). The numerical method used by Cai and Vijh (2005) bears two important limitations. First, it only can accomodate two state-variables (stock and market portfolio) and second, it is very computationally intensive. In contrast, our simulation-based algorithm can incorporate multiple state-variables, which is an interesting property in order to include shifts in the parameters like in Section 3.5 besides the market portfolio.

The rest of the chapter is organized as follows. Section 4.2 describes our executive and the simulation-based algorithm for the two state-variable case. Some numerical results are showed in Section 4.3. In Section 4.4 we perform a sensitivity analysis analyzing the effects of the dividend yield, the market risk-premium, the vesting period and the market correlation in the subjective ESO value, the expected exercise time, the subjective value of the restricted stock and the optimal portfolio allocation. We concentrate on the objective valuation in Section 4.5 using the exercise price thresholds. Section 4.6 study the moneyness effects and ESO incentives. Section 4.7 is about accounting implicantions. Finally, Section 4.8 concludes.

4.2 The model

As we mentioned above, in this section we show how to extend the LSMCU algorithm developed in Chapter 3 in order to consider the market portfolio as an alternative in the executive's investment opportunity set. We identify the executive's (subjective) ESO valuation using the certainty-equivalence, that is, the amount of cash delivered at the grant date optimally invested between the risk-free asset and the market portfolio until ESO maturity reporting the same expected utility to the executive. Like in Section 3.3, we first describe the executive's framework and secondly, we explain the modifications in the LSMCU method.

4.2.1 The executive framework

Following the notation from Chapter 3, we assume that the executive has an initial wealth W_0 of which a proportion α comprises restricted stocks of the company that can not be sold until ESO maturity. A higher value of α implies a less-diversified executive. The remainder of the non ESO executive's wealth, $(1-\alpha)W_0$ (outside wealth henceforth), is optimally invested between the market portfolio (with size η) and the risk-free asset (with size $1-\eta$). Moreover, the executive is granted with N_{eso} non-transferable American-style call options on the company stock with an exercise price of K and with a time to maturity of T years. Thus, conditional on non previous option exercise, the executive's wealth at maturity is

$$\begin{aligned} W_{T|T} &= W_0 \left[\alpha e^{dT} \frac{S_T}{S_0} + (1-\alpha) \left(\eta \frac{M_T}{M_0} + (1-\eta) e^{r_f T} \right) \right] + \\ & N_{eso} \left(S_T - K \right)^+ \end{aligned} \tag{4.1}$$

where $W_{i|j}$ indicates the executive's wealth at time i conditional on ESO exercise at time $j \leq i$, r_f is the risk-free interest rate, S_i and M_i are the stock price and the market portfolio price at time i respectively, d is the firm stock continuous dividend yield and the expresion $(b)^+$ represents the maximum between b and zero. From previous equation, note that we are assuming that the market portfolio does not pay dividends, that is, we suppose that the executive invests in the market portfolio through an index fund that reinvests all dividend proceeds. We also suppose that restricted stock dividend payments are reinvested in more restricted stocks. Also note that this setting is more general and nests the one in Hall and Murphy (2002) used in the previous chapter for $\eta = 0$ and $d = 0$.

If the executive decides exercising his ESOs at any time t prior to the maturity, we suppose he invests optimally the ESO payoffs until maturity. Then, his terminal wealth in the case of ESO exercise at time $t < T$ is

$$\begin{aligned} W_{T|t} &= \alpha W_0 \frac{S_T}{S_0} e^{dT} + \\ & (1-\alpha) W_0 \left[\eta \frac{M_T}{M_0} + (1-\eta) e^{r_f T} \right] + \\ & N_{eso} (S_t - K)^+ \left[\eta \frac{M_T}{M_t} + (1-\eta) e^{r_f (T-t)} \right]. \end{aligned} \tag{4.2}$$

Note that despite the ESO early exercise, the executive must keep his restricted stock until maturity and he splits the ESO payoffs between the market portfolio and the risk-free asset.

According with previous researches, we assume that firm stock price and the market portfolio value follow a joint geometric Brownian motion. This assumption implies that the stock price and the market portfolio follow a log-Normal distribution

$$\begin{aligned} S_{t+1} &= S_t \exp\left(\mu_s - \frac{1}{2}\sigma_s^2 - d + \sigma_s z_s\right) \\ M_{t+1} &= M_t \exp\left(\mu_m - \frac{1}{2}\sigma_m^2 + \sigma_m z_m\right) \end{aligned} \tag{4.3}$$

where μ_s and μ_m are the expected returns of the firm's stock and market portfolio respectively, σ_s and σ_m are the correponding volatilities, and z_s and z_m are the corresponding two correlated normal innovations with a correlation coefficient of ρ. We also adopt the CAPM as the model for the stock returns, that is, the expected rate of return satisfies

$$\mu_s = r_f + \beta\left(\mu_m - r_f\right) \tag{4.4}$$

where β is the systematic risk of the stock, i.e. $\beta = \rho\dfrac{\sigma_s}{\sigma_m}$. We consider a risk-averse executive with a power utility function with respect to his wealth described in equation (3.8).

The executive maximizes the expected utility of his terminal wealth allocating his outside wealth between the market portfolio (η^*) and the risk-free asset ($1 -$

η^*) and selecting his optimal ESO exercise time (t^*). Therefore, the executive's problem may be expressed as

$$\max_{\eta,t} \mathbb{E}_0 \left[U \left(W_{T|t} \right) \right]$$

As we mentioned above, solving the previous optimization program leads to obtain the optimal ESO exercise policy. This fact complicates the problem and some previous articles avoid this trouble considering only the risk-free asset as the alternative investment ($\eta^* = 0$) like in Kulatilaka and Marcus (1994) and Hall and Murphy (2002) or supposing the case of European style ESO which eliminates the problem of the optimal exercise rule ($t^* = T$), like in Tian (2004).

We calculate the ESO certainty-equivalence to obtain the subjective ESO valuation, that is, we find the amount of cash CE that provides the same expected utility than holding the ESO. Then, if an amount of cash equal to CE is delivered in place of the ESOs at grant date, the total executive's wealth at maturity is

$$\begin{aligned} W_T^{CE} &= \alpha W_0 e^{dT} \frac{S_T}{S_0} + \\ & [(1-\alpha)W_0 + N_{eso}CE] \times \left[\eta \frac{M_T}{M_0} + (1-\eta) e^{r_f T} \right] \end{aligned} \tag{4.5}$$

Obviously, the amount CE must be non restricted wealth that the executive may invest without restrictions between the market portfolio and the risk-free asset. Thus, to obtain the subjective value of the ESO we solve for CE the following expression according to equations (4.1) and (4.5)

$$\mathbb{E}_0 \left[U \left(W_T^{CE} \right) \right] = \mathbb{E}_0 \left[U \left(W_{T|t^*} \right) \right] \tag{4.6}$$

where the subindex in the expectation operator indicates the grant date. Specifically, we obtain the value of CE by a distance minimization like in equation (3.11).

We may also obtain the subjective valuation of the whole of the executive's wealth linked to the firm share price (restricted stock plus ESOs). In this case, the terminal wealth when an amount CE' is granted to the executive instead of the restricted stocks plus the ESOs is

$$W_T^{CE'} = \left[(1-\alpha)W_0 + CE'\right] \times \left[\eta \frac{M_T}{M_0} + (1-\eta)e^{r_f T}\right] \tag{4.7}$$

Once CE' is obtained, we can approximate per share the subjective valuation of the restricted stock as

$$S_R = \frac{CE' - N_{eso} \times CE}{N_s} \tag{4.8}$$

where N_s is the number of shares in the restricted stock package,

$$N_s = \frac{\alpha W_0}{S_0} \tag{4.9}$$

4.2.2 The utility-pricing algorithm

Following Cai and Vijh (2005), the algorithm consists on two stages. In the first stage we obtain the ESO exercise rule that maximizes the executive's expected utility, and in a second stage we determine the optimal portfolio allocation parameter, η^*.

Suppose we know the value of η^*, then it is straightforward to compute equation (4.5) for given simulated samples of the stock price S_T and the market portfolio M_T. To obtain $\mathbb{E}_0\left[U\left(W_{T|t^*}\right)\right]$ we adapt the LSMCU algorithm from previous chapter for the two state-variable case.

We start simulating m stock price paths and the correlated ones for the market portfolio under the real/physical measure according to equation (4.3). Let $\mathcal{U}$ be a vector of size $(m \times 1)$ where we shall collect the optimal utilities obtained from the ESO hold/exercise decision for every path. At each time step where the ESO is exercised, this vector will be updated with the maximizing utilities.

At maturity, the executive exercises the ESO if they are in the money. Therefore, if exercise is only allowed at maturity, $\mathcal{U}$ coincides with $U\left(W_{T|T}\right)$. One period before, at $T-1$, the executive has to decide between holding or exercising the ESO for all in the money paths. He will exercise (hold) the ESO when the expected utility of exercise becomes larger (smaller) than the expected utility of continue with the ESO for one period, that is, for those paths in the money at $T-1$, the ESO is exercised when

$$\mathbb{E}_{T-1}\left[U\left(W_{T|T-1}\right)|S_{T-1}, M_{T-1}\right] > \mathbb{E}_{T-1}\left[\mathcal{U}|S_{T-1}, M_{T-1}\right]. \tag{4.10}$$

To obtain the expected utility of the terminal wealth in case of exercise, $\mathbb{E}_{T-1}\left[U\left(W_{T|T-1}\right)|S_{T-1}, M_{T-1}\right]$, we first calculate $U\left(W_{T|T-1}\right)$ using equation (4.2) and the utility function. Second, we estimate the time $T-1$ expectation regressing $U\left(W_{T|T-1}\right)$ over some basis functions of the current state-variables, S_{T-1} and M_{T-1}. The projected value of the previous regression is the conditional expected utility, that is,

$$\mathbb{E}_{T-1}\left[U\left(W_{T|T-1}\right)|S_{T-1},M_{T-1}\right]=\widehat{U}\left(W_{T|T-1}\right) \tag{4.11}$$

The expected utility to continue one period with the ESO, $\mathbb{E}_{T-1}\left[\mathcal{U}|S_{T-1},M_{T-1}\right]$, is also computed by least-squares. In this case we regress the one period ahead utility, $\mathcal{U}$, over the same basis functions of the two state-variables. The projected values of this regression are the expected utilities when the executive continues with the ESO

$$\mathbb{E}_{T-1}\left[\mathcal{U}|S_{T-1},M_{T-1}\right]=\widehat{\mathcal{U}} \tag{4.12}$$

The exercise rule is obtained by comparison of equations (4.11) and (4.12) for each path in the money. For those paths where equation (4.10) is verified (the ESO is exercised), $\mathcal{U}$ is updated with the corresponding expected utility of exercise, $\widehat{U}\left(W_{T|T-1}\right)$. We can work backwards with this scheme. In general, at any exercise point t we have to compute $\mathbb{E}_t\left[U\left(W_{T|t}\right)|S_t,M_t\right]$ and $\mathbb{E}_t\left[\mathcal{U}|S_t,M_t\right]$ by least-squares. Once $\mathcal{U}$ has been updated until $t=0$ we solve numerically equation(4.6) for CE.

In the second stage, we use a grid search method to obtain η^*. We run the LSMCU procedure for a grid of values of η and we select the ESO value corresponding to the η value that maximizes the executive's expected utility. In Section 4.3.1 we describe more deeply this procedure and we illustrate it with a numerical example.

Note that this procedure leads a restriction in the portfolio optimization step. The executive is not allowed to rebalance his outside portfolio. This restriction supposes a suboptimal outside portfolio choice, which reduces the atractiveness of outside wealth. This imperfection also implies that the executive hold the same outside portfolio independently of the ESO exercise. Cai and Vijh (2005) also hold this restriction for computational reasons.

4.3 Numerical Study

In this section we show some numerical results from the application of the LSMCU algorithm over a hypothetical example. We first describe some numerical matters like the selected parameter values, simulations, etc.... Moreover, we show some numerical results not only about the ESO subjective value and also about the expected exercise time (L), the subjective value of the restricted stock (S_R) and the optimal portfolio allocation parameter (η).

4.3.1 Numerical Matters

Along this chapter, we use a general parameter set although we alter some of them in order to reveal some interesting findings about the subjective valuation. The executive has 5 million dollars in initial wealth (W_0) comprising company stock and safe cash. Moreover, he is granted with 25,000 ten-year American-style call options issued at the money, with an exercise price of 30. We take $\sigma_s = 0.40$, $\sigma_M = 0.20$, $\beta = 1$, $r_f = 6\%$ and $r_M - r_f = 0.05$. For the risk-aversion (γ) and the degree of diversification (α) we consider four different situations showed in

Table 4.1: Risk-aversion and diversification degrees.

	α	γ
case 1	0.5	2
case 2	0.5	3
case 3	0.75	2
case 4	0.75	3

This table shows the four different combinations of executive's risk-aversion (γ) and diversification degree (α) considered across this chapter.

Table 4.1[2].

Every result obtained with the LSMCU method is the average of one hundred previous estimations (50 seeds plus the 50 antithetics), and each estimation has been obtained running 200,000 price paths with 12 time steps per year (monthly frequency). Like in Chapter 3, we have observed that the model produces more accurate prices when enlarging the number of simulated paths instead of when increasing the frequency of simulations (weekly or daily), therefore we approximate the American-style ESO value with the subjective value of a Bermudan-style contract with 120 exercise points. In Figure 4.1 we show the convergence of the LSMCU method for different number of simulated paths. As we can see, the LSMCU method produces higher ESO subjective values when using a low number of paths in the simulations. However, when enlarging the number of paths, the ESO subjective value converges. As we explain in Section 3.4, we use as basis functions powers of $\log(S_t)$, $\log(M_t)$ and their cross product.

[2]These parameters are rather the same as in Cai and Vijh (2005)

As we have mentioned in Section 4.2.2, we run the LSMCU algorithm for a grid of values of η and we select the ESO subjective value as the one associated with the η value that maximizes the expected utility. Specifically, the procedure to obtain the correct ESO value is as follows:

1. We consider 11 points in the grid for η ranging from 0 to 1 with increments of 0.1. Thus, we run the LSMCU algorithm 11 times.

2. We approximate the relation between η and the maximum expected utility using a quadratic function defined as $\mathbb{E}\left[U(\eta)\right] = a\eta^2 + b\eta + c$ using the 11 pair of values.

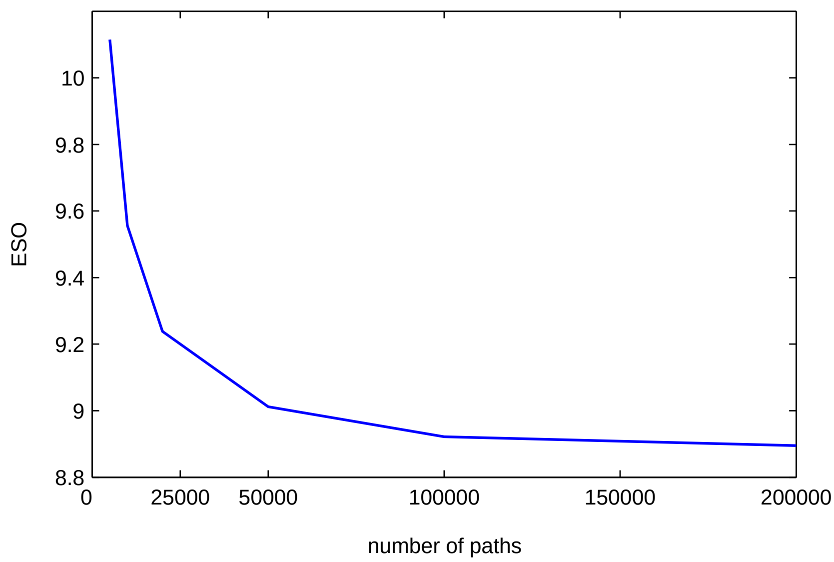
3. We obtain the optimal η as $\eta^* = -b/(2a)$.

4. We also approximate the relation between η and the ESO value (CE) as a quadratic function: $\text{ESO}(\eta) = a'\eta^2 + b'\eta + c'$.

5. Finally, we obtain the ESO value which maximizes the expected utility as $\text{ESO}(\eta^*) = a'\eta^{*2} + b'\eta^* + c'$.

To obtain the expected utility at grant date, Cai and Vijh (2005) use a modified version of the pyramidal rainbow grid of Rubinstein (1994) to accommodate the two state-variables. As we mention in Section 3.2, this method generates $(N+1)(N+2)(2N+3)/6$ nodes for N periods and they can only consider four exercise times per year. They run the rainbow grid for the different values of η ranging from 0 to 1 with increments of 0.01 and they select the η parameter that leads the higher expected utiliy. This procedure supposes 101 evaluations of the rainbow grid. Obviously, our polynomial approximation of the functions

Figure 4.1: Convergence of the LSMCU method

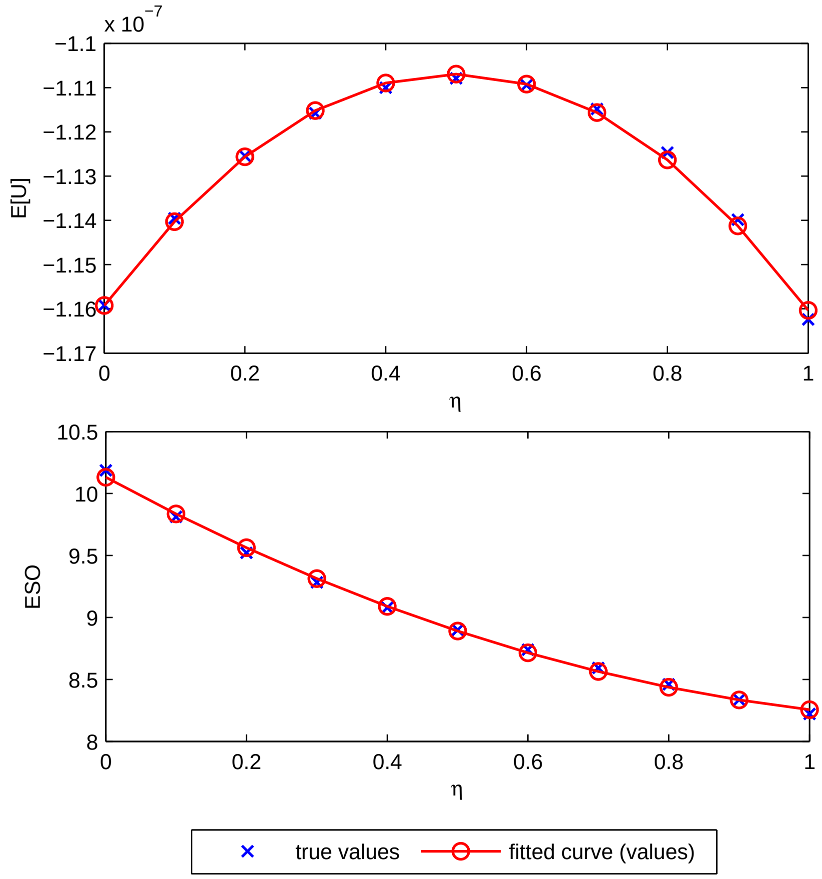


This figure shows the ESO price computed as the mean of 100 estimations (50 seeds plus 50 antithetics) for different number of simulated paths. We select $\{\alpha, \gamma\} = \{0.5, 2\}$. The remainder of the parameters are described in Section 4.3.1.

$\mathbb{E}[U(\eta)]$ and $\text{ESO}(\eta)$ is quite precise and much faster, since it only requires 11 evaluations of the LSMCU algorithm.

In Figure 4.2 we illustrate the main steps of the procedure described above for the case 1 in Table 4.1 and for our hypothetical executive and ESO plan. In the upper graphic we show the expected utility for each value of η obtained with the LSMCU algorithm (blue crosses) and the fitted quadratic curve (red line) with the corresponding fitted values (red circles). We may observe that the quadratic function is a good approximation and it exhibits a maximum. The more centered the crosses inside the circles, the better the (quadratic) approx-

Figure 4.2: Optimal portfolio and ESO value



The upper graphic shows the expected utility of an ESO plan with the features described in Section 4.3.1 for different values of the market portfolio weight (η) in the executive's outside wealth ($W_0(1-\alpha)$). The lower graphic shows the corresponding ESO subjective values. The blue crosses are the true values while the red line is the fitted quadratic polynomial. The red circles show the fitted values.

imation. Specifically, the fitted polynomial is $\mathbb{E}\left[U(\eta)\right] = -2.1122 \times 10^{-8}\eta^2 + 2.1008 \times 10^{-8}\eta - 1.1592 \times 10^{-7}$ and the maximum of this function is obtained with $\eta^* = 0.4973$. In the lower graphic we show the corresponding ESO values for each η value and the fitted curve and values also as a function of η. As we can see, the quadratic especification is also a good aproximations[3]. In this case, the fitted polynomial is $\text{ESO}(\eta) = 1.1245\eta^2 - 3.0906\eta + 10.132$. Thus, introducing η^* in this polynomial we obtain the ESO value that maximizes the expected utility with the best portfolio allocation, $\text{ESO}(\eta^*) = 8.895$.

We would like to remark an interesting finding from Figure 4.2. Observe that the η value that maximizes the executive's expected utility ($\eta^* = 0.4973$) is not the η value that maximizes the subjective value of the ESO. Looking at the bottom graphic, taking values of $\eta < \eta^*$, the subjective ESO value increases, however, this would suppose a decrease in the executive's expected utility (upper graphic). Remember that the problem we try to solve is to achive the maximum expected utility and not to find the maximum subjective ESO value.

4.3.2 The ESO subjective value

We show numerical results about the subjective ESO value, the expected exercise time, the outside portolio composition and the restricted stock subjective valuation in Table 4.2. We consider two values for the risk-aversion coefficient, $\gamma = \{2, 3\}$ and five levels of executive's diversification degree, $\alpha = \{0.05, 0.25, 0.50, 0.75, 0.95\}$[4]. Note that these parameter sets include the ones

[3]In both quadratic approximations the R^2 statistic of the regressions is higher than 0.99.
[4]In Section 4.4.1 we consider a wider range of values for γ.

from the general cases in Table 4.1, and we also consider the case of $\alpha = 0.25$ and the two limit cases of $\alpha = 0.05$ and $\alpha = 0.95$ which correspond to a well diversified and an extremely bad diversified executive respectively. For comparison reasons, we include in Table 4.2 the Black-Scholes value of the ESO. Since the stock does not pay dividends, this value coincides with the corresponding American-style option price. Below each value in Table 4.2, we display in parenthesis the standard deviation computed over the 100 estimates (from the 50 seeds plus the 50 antithetics).

ESO value

The main result of Table 4.2 is that the executive's subjective valuation may suppose a very high discount over the risk-neutral valuation depending on the executive's risk-aversion and diversification degree: the less diversified (higher α) and/or more risk-averse (higher γ) the executive, the lower ESO subjective value. A higher value of α implies that the executive's wealth contains a larger proportion of firm stock. In consequence, the executive supports more specific risk and he discounts stronger the ESO value, i.e. for $(\alpha, \gamma) = (0.25, 2)$ the subjective ESO value is 11.292, meanwhile it becomes 9.764 for a more risk-averse executive ($\gamma = 3$) and 6.806 for a less diversified executive ($\alpha = 0.75$). The European-style ESO shows the same pattern but these values become lower for high values of α, that is, a lower diversified executive discounts strongly the European-style ESO, since he supports a lot of specific risk and he cannot obtain the ESO payoffs until maturity (10 years). Also note that, in general, the standard deviations produced by the LSMCU algorithm are quite low, which is a desirable property.

Expected exercise time

In row L we show the expected exercise time of the ESO (in years). L has been computed as the mean of the exercise times over the 200,000 paths and over the 100 estimates. Note that L shows the same behaviour than ESO and ESO_E. That is, the expected exercise time decreases with the executive's lack of diversification and risk-aversion coefficient. This result suggests that ESO objective value[5] (the firm ESO cost) will be lower than the risk-neutral one since the early exercise is not the optimal exercise rule under the risk-neutral measure for a call option on a stock with non dividend payments. We will analyze the objective valuation in Section 4.5

Portfolio decisions and restricted stock

The row η^* displays the percentage of outside wealth invested in the market portfolio. We observe that the more risk-averse the executive becomes, the lower the percentage of outside wealth invested in the market portfolio (and the higher the one invested in the risk-free asset). When the executive becomes more risk-averse, he prefers a new portfolio with a larger proportion of the risk-free asset in order to reduce his risk exposure. Also observe how η value depends on the diversification degree α. Finally, the row labelled S_R reports the subjective

[5]Remember that the objective ESO value is the valuation from a risk-neutral agent point of view but considering that exercise decisions are made by a restricted and undiversified agent.

Table 4.2: Executive's behaviour: ESO value, restricted stock, exercise time and wealth allocation

	α				
$BS = 18.776$	0.05	0.25	0.50	0.75	0.95
Panel A: $\gamma = 2$					
ESO	14.211	11.292	8.895	6.806	4.467
	(0.06)	(0.06)	(0.07)	(0.07)	(0.06)
ESO_E	11.688	7.148	4.031	1.900	0.514
	(0.05)	(0.03)	(0.02)	(0.01)	(0.00)
L	8.522	7.629	6.670	5.366	3.714
	(0.05)	(0.06)	(0.06)	(0.07)	(0.06)
S_R	27.438	22.719	19.001	15.808	13.377
	(0.22)	(0.08)	(0.06)	(0.05)	(0.04)
η^*	0.594	0.554	0.497	0.388	0.052
Panel B: $\gamma = 3$					
ESO	13.121	9.674	7.328	5.408	3.036
	(0.04)	(0.05)	(0.06)	(0.05)	(0.03)
ESO_E	9.818	4.772	1.990	0.579	0.052
	(0.04)	(0.02)	(0.01)	(0.00)	(0.00)
L	7.967	6.842	5.556	4.144	2.550
	(0.03)	(0.04)	(0.04)	(0.05)	(0.04)
S_R	27.236	20.934	16.495	12.710	8.772
	(0.17)	(0.06)	(0.04)	(0.03)	(0.03)
η^*	0.372	0.330	0.277	0.195	0.001

This table shows the subjective ESO value, the corresponding European ESO value (ESO_E), the expected exercise time (L), the value per share of the restricted stock (S_R) and the (optimal) proportion of outside wealth invested in the market portfolio (η^*) for 5 different coefficients of the diversification degree (α) and for two risk-aversion levels (Panel A and Panel B). The parameter α is the percentage of total wealth invested in restricted stock. BS denotes the Black-Scholes price of the ESO. Below each value, in parenthesis, we display the standard deviation computed over the 100 estimates. The remainder of the parameters are described in Section 4.3.1.

value per share of the restricted stock following equation (4.8). According to previous results, S_R also decreases with α and γ. Maintaining the non option wealth constant, the more restricted stock is granted to an executive (higher α), the lower his incentives (per unit of stock), since the stock is less valued by the executive.

One state-variable vs. two state-variable framework

In order to show the importance of considering two state-variables ($s2$) instead of the one state-variable framework ($s1$), in Table 4.3 we report the values for ESO, ESO_e, L, and S_R for $\eta = 0$. In this table, in parenthesis, we display the relative bias of the one state-variable case with respect to the results under the two state-variable framework in Table 4.2. The bias has been calculated as $(ESO_{s1} - ESO_{s2})/ESO_{s2}$. The main finding is that not consider the market portfolio in the executive's wealth supposes a possitive bias in all four variables, i.e. for $(\alpha, \gamma) = (0.25, 2)$ the subjective ESO value is 14.275 which supposes an overvaluation around 26%, the bias for the European-style ESO is around 25%, the expected life bias is 7% and the bias for the subjective value of the restricted stock is 39%. In the one state-variable framework, the executive invests all his outside wealth in the risk-free asset, thus the higher expected return provided by the restricted stocks and the ESOs is more wellcome than in the two state-variable framework, where the executive already has a significant proportion of his wealth invested in the (risky) market portfolio. In this situation, the discount due to the idiosyncratic risk is higher than the benefits from the larger expected return.

Moreover, we can appreciate that the relative biases decreas with the executive's lack of diversification, for $\alpha = 0.95$ the results for both frameworks are rather the same. We also can observe some inconsistent values in the one state-variable framework[6], i.e. for $(\alpha, \gamma) = (0.05, 2)$ the subjective ESO value is 20.269, which is larger than the risk-neutral valuation, 18.776 (Black-Scholes price). Furthermore, for the same situation, the subjective value per share of the ten-years restricted stock is 42.601, while the current market value is $S_0 = 30$.

4.4 Sensitivity analysis

In this section we extend the analysis of the subjective ESO valuation considering different values for the risk-aversion, dividends, vesting period, market risk-premium and the market-β. Specifically, we show how the subjective ESO value, the expected exercise time, the optimal portfolio allocation and the subjective value of restricted stock change when altering the previous parameters.

4.4.1 The risk-aversion

Friend and Blume (1975) suggest that investors exhibit a relative risk-aversion coefficients of between 2.5 and 4.0, and Litzenberger and Ronn (1986) estimate a value close to 4.0. Thus, in Figure 4.3 we consider values for the relative risk-aversion coefficient ranging from 1.5 to 4.5 in order to show the behaviour of some outputs of the LSMCU algorithm. According to previous results, we

[6] As we mention in the Introduction, Cai and Vijh (2005) also found these inconsistent values.

Table 4.3: Two state-variable vs. one state-variable

	α				
$BS = 18.776$	0.05	0.25	0.50	0.75	0.95
		Panel A: $\gamma = 2$			
ESO	20.269	14.275	10.186	7.304	4.490
	(0.43)	(0.26)	(0.15)	(0.07)	(0.01)
ESO_E	18.450	9.899	5.046	2.148	0.518
	(0.58)	(0.38)	(0.25)	(0.13)	(0.01)
L	9.271	8.129	7.023	5.487	3.725
	(0.09)	(0.07)	(0.05)	(0.03)	(0.00)
S_R	42.601	31.620	24.750	19.259	13.865
	(0.55)	(0.39)	(0.30)	(0.22)	(0.04)
		Panel B: $\gamma = 3$			
ESO	17.560	11.395	7.960	5.645	3.037
	(0.34)	(0.18)	(0.09)	(0.04)	(0.00)
ESO_E	14.565	6.273	2.394	0.637	0.052
	(0.48)	(0.31)	(0.20)	(0.10)	(0.00)
L	8.677	7.274	5.711	4.261	2.553
	(0.09)	(0.06)	(0.03)	(0.03)	(0.00)
S_R	40.025	27.382	20.103	14.472	8.842
	(0.47)	(0.31)	(0.22)	(0.14)	(0.01)

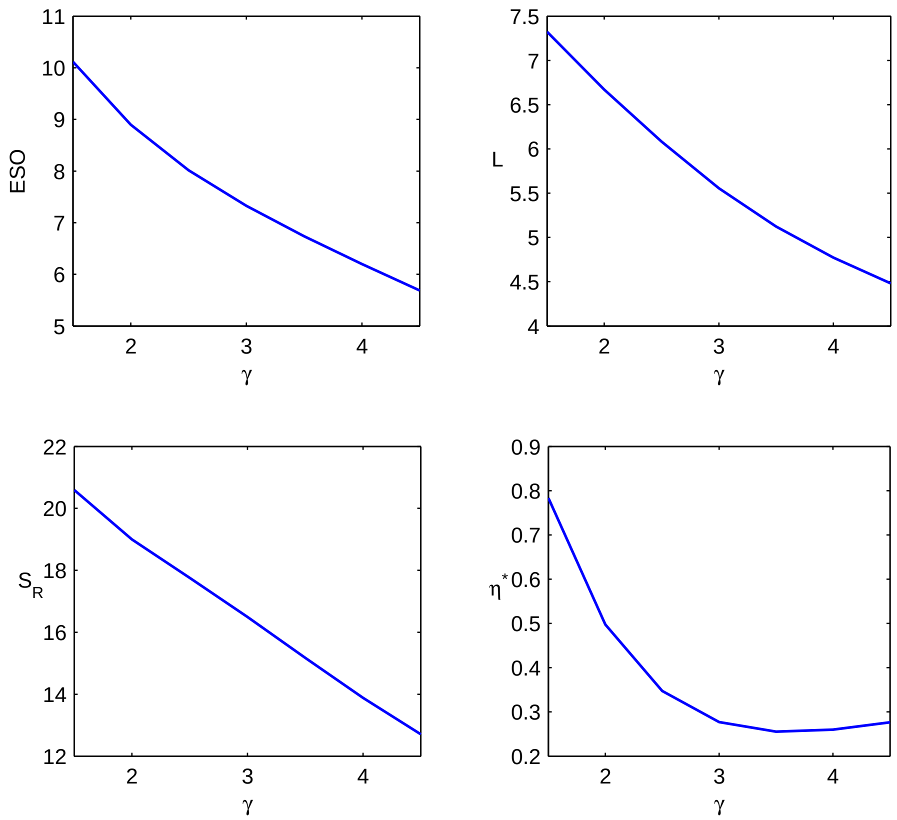
This table shows the subjective ESO value, the corresponding European ESO value (ESO_E), the expected exercise time (L) and the value per share of the restricted stock (S_R) in the one state-variable framework ($s1$), for 5 different coefficients of the diversification degree (α) and for two levels of risk-aversion (Panel A and Panel B). The parameter α is the percentage of total wealth invested in restricted stock. BS denotes the Black-Scholes prices of the ESO. Below each value, in parenthesis, we display the relative bias with respect to the values under the two state-variable framework ($s2$) from Table 4.2 calculated as $(ESO_{s1} - ESO_{s2})/ESO_{s2}$. The remainder of the parameters are described in Section 4.3.1.

observe a decreasing pattern for the four variables when increasing the risk-aversion coefficient. The more risk-averse the executives are, the lower both the ESO and the restricted stock values. Furthermore, it holds that the expected exercise time diminishes with γ and the market portfolio size (in the executive's portfolio), becomes smaller.

4.4.2 The dividends

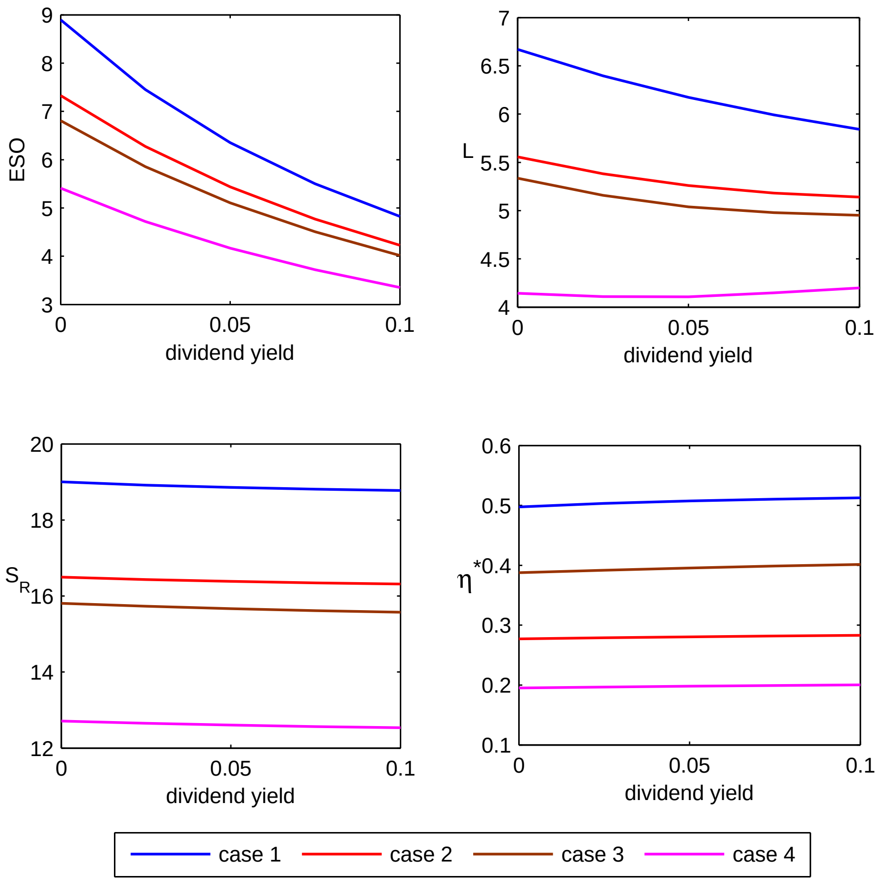
In Figure 4.4 we consider different firm stock dividend yields ranging from 0 to 0.1. A positive dividend yield supposes a lower drift for the stock price process. Therefore, the ESO value will fall respect to the non dividend payment case. This fact may be observed in the upper-left graphic for all pairs (α, γ) reported in Table 4.1. The expected exercise time also shows a decreasing behaviour, except for the case 4 (more risk-averse and less diversified) where it is flat. However, the subjective value of the restricted stock remains approximately constant (bottom-left graphic). According to Section 4.2.1 stock dividend payments are reinvested in firm stocks (see equations (4.1) and (4.2)), thus dividend payments should not have effects on the subjective valuation of the restricted stock. Finally, the proportion of outside wealth invested in the market portfolio remains constant for the four cases.

Figure 4.3: Risk-Aversion and subjective valuation



This Figure shows the behaviour of the subjective ESO value, the expected exercise time, the subjective value of the restricted stock and the optimal portfolio composition for different levels of executive's risk-aversion ranging from 1.5 to 4.5. We consider that a half of the executive non-option wealth is restricted stock ($\alpha = 0.5$). The remainder of the parameters are those described in Section 4.3.1.

Figure 4.4: Dividends and subjective valuation



This Figure shows the behaviour of the subjective ESO value, the expected exercise time, the subjective value of restricted stock and the market portfolio for different dividend yields, ranging from 0 to 0.10. We consider the four cases described in Table 4.1. The remainder of the parameters are those described in Section 4.3.1.

4.4.3 The vesting period

Now, we analyze the effects of the vesting period. As we mention in previous chapters the vesting period is one of the main differences between tradable (conventional) options and ESOs, thus to asses the impact of this characteristic in the executive's ESO valuation and its exercise behaviour becomes relevant. In Figure 4.5 we consider a grid for the vesting period ranging from 0 years (American-style ESO) to 10 years (European-style ESO) to show the behaviour of the four outputs of the LSMCU algorithm. As expected, the larger the vesting period, the lower the ESO subjective value, since the executive has less opportunities to obtain cash from the ESO exercise. Obviously, the expected exercise time increases with the vesting period.

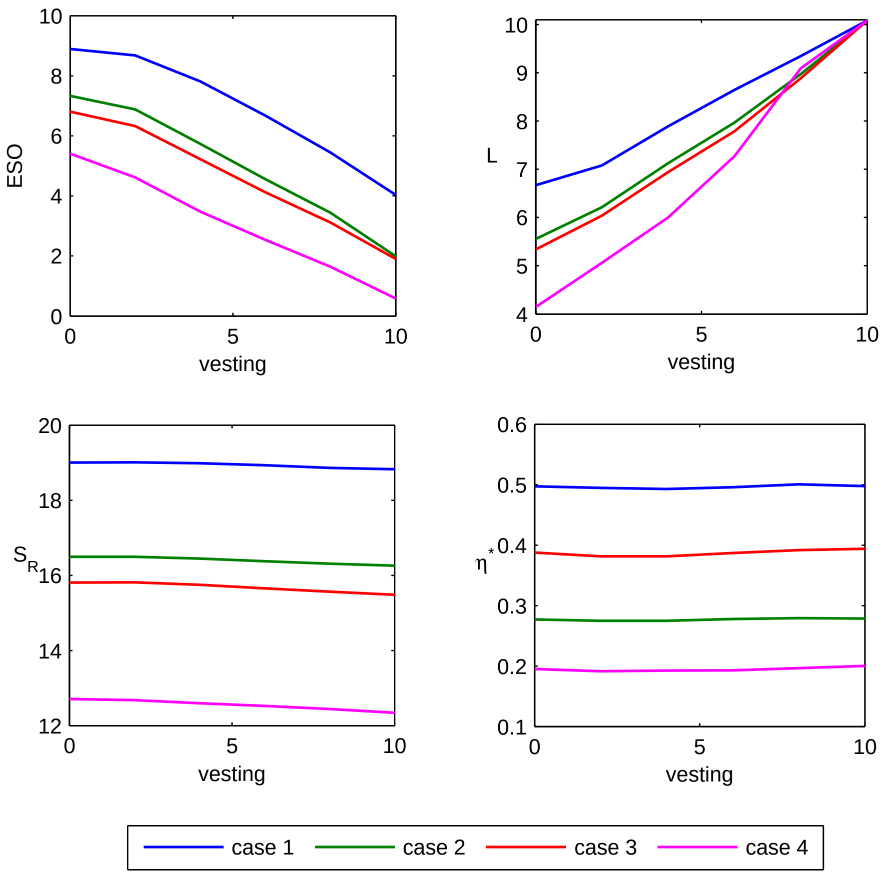
The vesting period length has not relevant effects on the restricted stock, S_R, since it exhibits a very slow decreasing pattern. The optimal exercise time is delayed when introducing a vesting period, then the executive holds more firm specific risk (ESOs plus stocks) more time. In this situation, the executive discounts all firm performance related assets. Finally, like in the dividend analysis, the proportion of outside wealth invested in the market portfolio remains constant.

The previous analysis suggests that the variables that really affect the executive's portfolio composition (η) are the risk-aversion (γ) and diversification degree (α), and not the dividend yield or the vesting period.

4.4.4 The market risk-premium

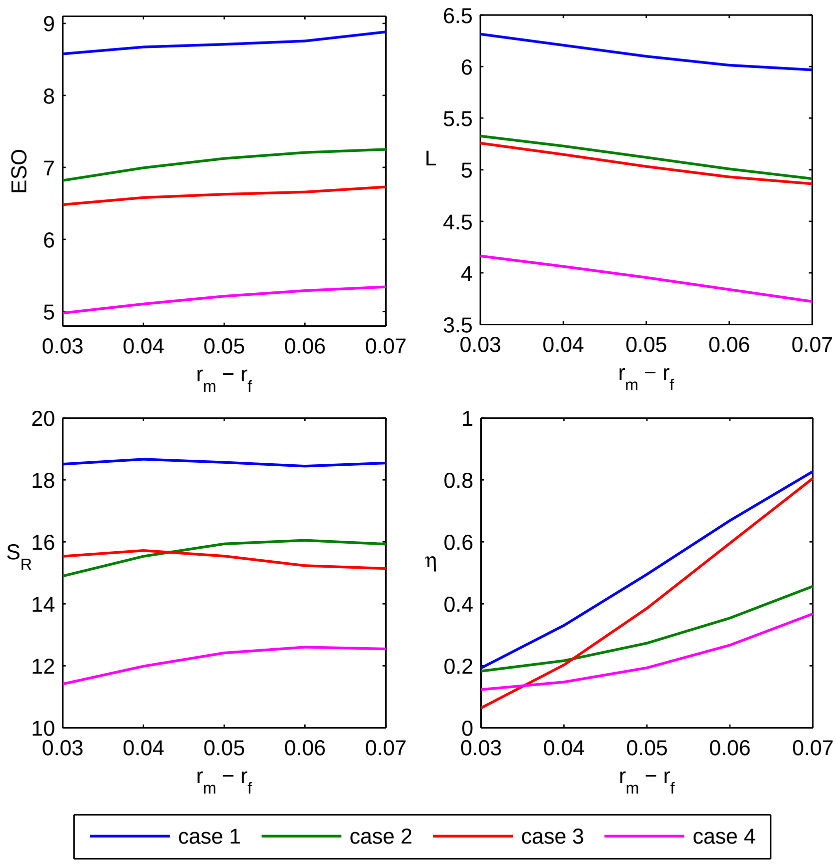
In contrast with the risk-neutral framework, where the drift of the price process is irrevelant to value options, in the utility-based model the expected stock rate of return has important effects. In this section we study the behaviour of the four outputs of the LSMCU algorithm considering different values for the market risk-premium ($r_m - r_f$) ranging from 0.03 to 0.07. Figure 4.6 shows the results for the four outputs of the LSMCU algorithm. The main findings are, first, the subjective ESO value increases with the risk-premium for all cases, as expected. Secondly, the expected exercise time reduces for higher market risk premium. A higher market risk-premium produces either higher stock and market portfolio prices, therefore the executive will exercise the ESOs earlier in order obtain the ESO payoffs and mitigate his lack of diversification, despite possible upward future movements of the stock price. Third, the restricted stock value, S_R, exhibits a clear increasing pattern only for the cases 2 and 4 which correspond to a more risk-averse agent ($\alpha = 0.75$). A more risk-averse executive is more sensitive to the risk-return tradeoff. Thus, an increase in the expected rate of return of the stock will have more effects in the subjective valuation of stocks for executives with higher risk-aversion. Finally, the higher the market risk-premium, the larger the size of outside wealth invested in the market portolio (η). An increase in the market risk-premium supposes more compensation for bearing the same quantity of systematic risk. Then, the executive will prefer a higher proportion of the market portfolio since now it becomes more atractive. This finding is more intense (higher slope) for less risk-averse executives (cases 1 and 3, $\gamma = 2$) than for the more risk-averse ones (cases 2 and 4, $\gamma = 3$) because a large risk-aversion penalize stronger the positive effects of the higher market risk-premium.

Figure 4.5: Vesting period and subjective valuation



This Figure shows the behaviour of the subjective ESO value, the expected exercise time, the subjective value of restricted stock and the market portfolio for different vesting period lengths, ranging from 0 to 100% of the life of the ESO. We consider the four cases described in Table 4.1. The remainder of the parameters are those described in Section 4.3.1.

Figure 4.6: Market risk premium and subjective valuation



This Figure shows the behaviour of the subjective ESO value, the expected exercise time, the subjective value of restricted stock and the market portfolio for different market risk-premiums, ranging from 0.03 to 0.07. We consider the four cases described in Table 4.1. The remainder of the parameters are those described in Section 4.3.1.

4.4.5 The market-β

We also analyze the impact of the correlation between the firm stock and market portfolio returns in the executive's behaviour and ESO valuation. The market correlation of the firm stock returns is measured through the β since $\beta = \rho \frac{\sigma_s}{\sigma_m}$, as we have mentioned previously. According with the CAPM, an increase (decrease) of β supposes a higher (lower) stock rate of return, see equation (4.4). Moreover a change in β also yields a different risk composition. Following the CAPM, the total variance of the stock returns can be decomposed in two terms as follows

$$\sigma_s^2 = \beta^2 \sigma_m^2 + \sigma_{\varepsilon_s}^2 \tag{4.13}$$

where $\beta^2 \sigma_m^2$ and $\sigma_{\varepsilon_s}^2$ are the systematic and the idiosyncratic risk of the firm stock respectively. Thus, if we mantain constant the level of total risk σ_s^2, an increase (decrease) in β also supposes a higher (lower) proportion of systematic risk. Note that this analysis is different from the one conducted in the previous section since a higher (lower) market risk-premium supposes a larger (smaller) expected rate of return for either the stock and the market portfolio without changes in the stock variance composition.

In Table 4.4 we show the differences, in percentage, of the outputs of the LSMCU method for two different levels of β (panel A and panel B) with respect to the general case showed in Table 4.2 ($\beta = 1$). As expected, a lower β supposes a decrease in ESO and S_R, since a fall in β implies a lower expected rate of return, i.e. for $(\alpha, \gamma) = (0.75, 3)$ ESO and S_R drop till -10.558% and till -19.599% respectively for $\beta = 0$. Moreover, the executive delays the ESO exercise in order

Table 4.4: Market-β and subjective valuation

	(α, γ)			
	(0.5, 2)	(0.5, 3)	(0.75, 2)	(0.75, 3)
Panel A: $\beta = 0$				
ESO	-8.510	-8.570	-10.212	-10.558
L	5.502	11.609	10.827	17.375
S_R	-12.541	-16.829	-16.726	-19.599
η^*	60.966	94.946	132.47	209.74
Panel B: $\beta = 0.5$				
ESO	-6.914	-7.096	-8.728	-8.561
L	2.024	5.274	4.771	6.346
S_R	-10.863	-15.162	-14.986	-18.190
η^*	36.821	47.292	84.278	101.03

This table shows the percentage differences of the subjective ESO value, the expected exercise time, the restricted stock value and the optimal portofolio allocation parameter with respect to the values for the case showed in Table 4.2 ($\beta = 1$) for different levels of diversification and risk-aversion and for two different values of β in panels A and B. The remainder of the parameters are described in Section 4.3.1.

to obtain enough profits, i.e. L increases till 17.375% for the same situation. Finally, η^* changes widely, it increases up to 209%, that is, the executive duplicates the wheight of the market portfolio on his own portfolio. These results hold for $\beta = 0.5$ (Panel B) although the size of the percentage differences are lower.

4.5 The objective value

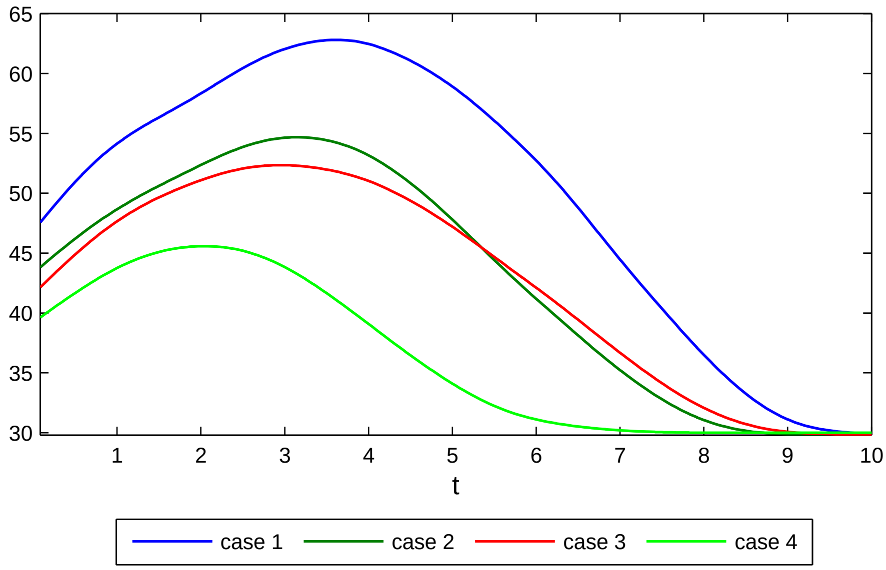
As we mention in Chapter 3, the objective ESO value is the cost to the firm of issuing ESOs, and this value may be approximated considering the risk-neutral price process but using the exercise policy made by the executive obtained under the real/physical measure with the LSMCU algorithm. Specifically, in this section we obtain the objective ESO value using the exercise price thresholds obtained with the LSMCU method like in Section 3.4.2.

4.5.1 The ESO firm cost

Figure 4.7 displays the exercise price thresholds implied by our hypothetical executive and ESO grant. Each line corresponds to a different (α, γ) pair according to Table 4.1. The price thresholds are the time-varying frontiers where the executive is indifferent between holding or exercising the ESO. Above the threshold the executive will choose to exercise the ESOs while below the threshold he will hold them. These thresholds have been calculated taking the average of the minimum stock prices at which the ESO is exercised for each time step and also for the 100 estimations (50 seeds plus 50 antithetics). Finally, the thresholds showed in Figure 4.7 have been smoothed using cubic splines. We observe that these price thresholds exhibit the same behaviour than in the one state-variable framework, the price thresholds become lower either for more risk-averse and for less diversified executives.

In Table 4.5 we display for each pair (α, γ) from Table 4.1 the risk-neutral

Figure 4.7: Price thresholds



This Figure shows the price thresholds, S^*, for the four cases in Table 4.1. For each date t, S^* is the average (over the 100 estimates) of the minimum price for which the executive exercises the ESO. The thresholds have been smoothed using cubic splines. For each case, above the price threshold the executives exercises the ESO, meanwhile he holds the ESO below the threshold. The remainder of the parameters are those described in Section 4.3.1.

Table 4.5: ESO objective value and executive's discount

	(α, γ)			
	(0.5, 2)	(0.5, 3)	(0.75, 2)	(0.75, 3)
Risk-Neutral value	18.047	18.047	18.047	18.047
Objective value	12.739	11.121	10.804	8.843
Exec. Discount (%)	30.177	34.109	37.007	38.841

This table shows the risk-neutral ESO value, the objective ESO value, and the executive's ESO value discount for the four cases described in Table 4.1. The risk-neutral value is obtained through the Black-Scholes formula, the objective one using the exercise price thresholds displayed in Figure 4.7 and the executive's ESO value discount has been calculated as $(ESO_{obj} - ESO_{sub})/ESO_{obj} \times 100$ where ESO_{sub} is the subjective ESO valuation displayed in Table 4.2. The remainder of the parameters are those described in Section 4.3.1.

ESO value, the objective value (firm cost) of the ESO using the price thresholds from Figure 4.7 and the executive's ESO discount in percentage, computed as $(ESO_{obj} - ESO_{sub})/ESO_{obj} \times 100$ where ESO_{sub} is the subjective ESO valuation displayed in Table 4.2. The objective ESO values have been obtained taking the mean over 100 estimations (50 seeds plus the 50 antithetics) and running 20,000 paths for each estimation[7].

The main finding in Table 4.5 is that the objective value is lower than the ESO risk-neutral valuation. This fact suggests the well-knwon idea of BS overvaluation when accounting for ESO cost. Further, the executive's ESO discount is significant, higher than 30% with respect to the objective value (firm cost), and it supposes that the whole of the ESO cost is not perceived as an incentive by the executive, that is, a firm may be expending some funds in a compensation

[7]In this case we only consider 20,000 paths intead of 200,000 because we are valuing a barrier option under the risk-neutral measure without the need to apply any algorithm to determine the exercise rule. Therefore, we can reduce the number of paths in order to increase the speed of the algorithm without a significant loss of precission.

package that is strongly discounted by the executive. For managerial purposes, it would be interesting to find the compensation contract with the lower executive's discount.

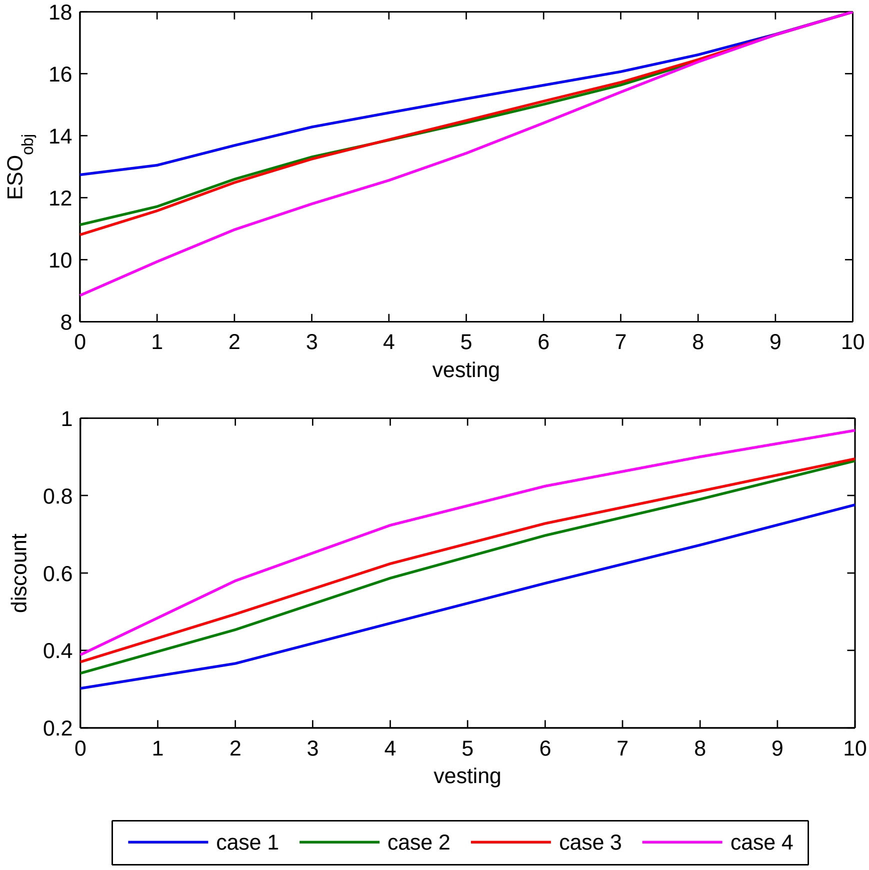
4.5.2 Vesting protection

In Figure 4.8 we show the objective ESO value (upper graphic) and the executive's discount (bottom graphic) calculated as $\frac{ESO_{obj}-ESO_{sub}}{ESO_{obj}}$ for different vesting period lengths ranging from 0 (American-style ESO) to 10 (European-style ESO). Each line in both graphics represents a pair (α, γ) from Table 4.1.

In the upper graphic of Figure 4.8 we can appreciate the so-called vesting protection phenomenon: the less opportunities to exercise the ESO, the higher the ESO firm cost (objective value) becomes. As we explain in Section 3.4.3, a larger vesting period "protects" the executive from his own suboptimal (from the risk-neutral point of view) exercise decision and it produces an increase in the ESO cost, despite the fall in the executive's utility.

The executive's discount (bottom graphic) is interesting for compensation packages design since it indicates the difference between both values, the firm ESO cost and the executive's incentive. Thus, according to Figure 4.8, ESOs with shorter vesting periods are more attractive for executives and the associated firm costs are lower compared with ESOs with a larger vesting period. We also observe that the ESO discount is higher as more risk-averse or less diversified becomes the executive, i.e. for a vesting period of 4 years, the ESO discount for a case 1 executive is around 47%, meanwhile the discount increases up to 72%

Figure 4.8: Objective value and vesting period



This Figure shows in the upper graphic the ESO objective value using the exercise price thresholds for the four cases in Table 4.1 and for different vesting periods. The bottom graphic displays the executive's ESO discount calculated as $(ESO_{obj} - ESO_{sub})/ESO_{obj}$ where ESO_{sub} shows the subjective ESO values displayed in Figure 4.5. The remainder of the parameters are those described in Section 4.3.1.

Table 4.6: Moneyness and ESO incentives

		(α, γ)			
	BS	(0.5, 2)	(0.5, 3)	(0.75, 2)	(0.75, 3)
itm ($K = 25$)	0.8945	0.6396	0.6159	0.6126	0.6089
atm ($K = 30$)	0.8658	0.5579	0.4851	0.4702	0.4529
otm ($K = 35$)	0.8377	0.4995	0.4420	0.4222	0.3644

This table shows the numerical subjective *deltas* at grant date for three different moneyness situations and for different levels of risk-aversion and degrees of diversification (see Table 4.1). The stock price at grant date is $S_0 = 30$ in all cases. The remainder of the parameters are those described in Section 4.3.1.

for a case 4 executive.

4.6 Incentives and Moneyness

In this section we analyze the incentive effects of ESOs and we focus on how the ESO subjective value changes in response to stock price movements. According to Hall and Murphy (2000, 2002) and Calvet and Rahman (2006), among others, incentives are defined as the change in the certainty-equivalence value of ESO holdings for a dollar change in the stock price. This measure corresponds to the subjective *delta* of the ESO[8]. The higher the ESO's *delta*, the larger the executive profits from an increase in the underlying stock price, and therefore the higher the incentives to undertake actions to increase the stock price.

[8]Johnson and Tian (2000) and Jorgensen (2002) also use the ESO *delta* to measure de incentive effects in a risk-neutral framework. In our case, $delta = \frac{\partial CE}{\partial S_0}$, and it has been computed numerically perturbating S_0.

In Table 4.6 we display the subjectives *deltas* for the four pairs $\{\alpha, \gamma\}$ and the corresponding Black-Scholes *deltas* in column BS for comparison. We also consider three different moneyness situations altering the ESO exercise price. The main findings are the followings. First, the subjective *deltas* are lower than the risk-neutral ones. Moreover, the difference between risk-neutral and subjectives *deltas* becomes larger for out of the money (otm henceforth) options. This means that otm ESOs are very discouraging for risk-averse and undiversified executives, i.e. the Black Sholes *delta* for the otm option is 0.8377 while it drops till 0.3644 for an executive with $(\alpha, \gamma) = (0.75, 3)$. Second, the more undiversified and risk-averse the executive, the lower the ESO incentives (lower *deltas*). Finally, ESOs issued in the money provide more incentives than when they are granted at the money or out of the money.

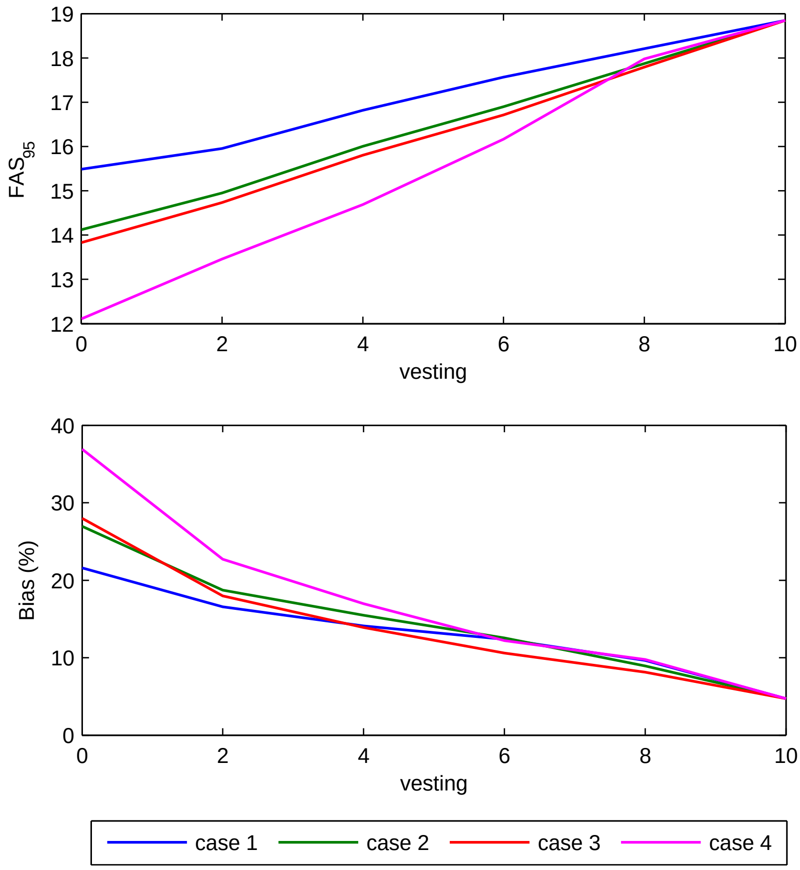
4.7 Accounting implications

As we have discussed in Section 2.6, in 1995 the Financial Accounting Standard Board (FASB) issue the Financial Accounting Standadard 123 (FAS 123) which encourages firms to account ESO grants using the fair value. This fair value is the valuation made by a risk-neutral agent (the firm) but recognizing the early exercise feature of ESO. The FAS 123 suggests to use the Black-Scholes formula but changing the time to maturity of the ESO by its expected life in order to taking into account the early exercise. Moreover, this value must be corrected by the probability of executive's departure during the vesting period.

The upper graphic of Figure 4.9 displays the ESO cost according with the

FAS 123 proposal following equation (2.18), for the four cases in Table 4.1 and for different vesting periods. As we can see, the FAS 123 ESO cost increases with the vesting period since a larger vesting period implies a larger expected ESO life, according to the findings from Figure 4.5. The bottom graphic shows the relative biases in percentage of the FAS 123 method with respect to the correct ESO costs (objective value) displayed in Figure 4.8. The biases are computed as $(FAS_{95} - ESO_{obj})/ESO_{obj} \times 100$. The main findings are that the FAS 123 method overprices the ESO cost. This overvaluation is significant and higher (lower) for shorter (larger) vesting periods (larger than 30%). Secondly, for shorter vesting periods (up to 4 years, which in practice are the most common situations) the bias size is larger when the executive becomes more risk-averse and/or less diversified.

Figure 4.9: FAS 123 and firm ESO cost



This Figure shows in the upper graphic the ESO fair value according with the FAS 123 proposal in equation (2.18) for the four cases in Table 4.1 and for different vesting periods. The bottom graphic displays the relative biases in percentage calculated as $(FAS_{95} - ESO_{obj})/ESO_{obj} \times 100$ where ESO_{obj} are the objective ESO values displayed in Figure 4.8. The remainder of the parameters are those described in Section 4.3.1.

4.8 Concluding Remarks

In this chapter we show how to combine simulations with the certainty-equivalence framework to solve the subjective ESO valuation problem. Since our model is based on simulations, it is suitable to deal with multiple state-variables (stock price and market portfolio) and allowing early exercise (American-style ESO). We show how the ESO subjective value is negatively related with the executive's risk-aversion and his diversification degree. We also account for the the vesting period effects, revealing that the shorter the vesting period, the higher the subjective valuation. Moreover, we report results about the expected exercise time, the subjective restricted stock value and the proportion of outside wealth invested in the market portfolio. All these variables also diminish with the risk-aversion and the lack of diversification. We also obtain significant possitive differences when consider the simpler one state-variable framework. In this case, under some parameters, the model produces inconsistent subjective values for both the ESO and restricted stock.

We also conduct a sensitivity analysis showing how changes in the dividend yield, vesting period, market risk-premium and β have effects on the subjective valuation of ESO and restricted stock, and in the executive's behaviour. Dividens, vesting period and β reduce the subjective ESO value while the market risk-premium has a possitive effect.

We study the objective ESO value (firm cost) using the exercise price thresholds obtained in the subjective valuation. We find that the objective value is lower than the risk-neutral valuation for all cases considered suggesting the Black-

Scholes overvaluation when accounting for ESO cost. Moreover, the objective value increases with the vesting period length, just the opposite of the subjective value does, showing the vesting protection phenomenon. In consequence, the executive discounts stronger ESO values for long-term vested ESO, reducing its attractiveness.

We analyze the ESO incentives, measured as the subjective delta for different moneyness situations. The subjective delta is negatively related with the executive's risk-aversion and lack of diversification. Moreover, the subjective delta also varies with the ESO moneyness: ESOs granted in the money have a higher delta providing more incentives, than at or out the money ESOs. Moreover, we report numerical evidence of the FAS 123 (1995) overvaluation (biases higher than 30%) when accounting for ESO cost, using the expected exercise time implied in the LSMCU model instead of the true ESO cost (objective value).

Finally, further research should be to extend the simulation-based method to include the cases of indexed executive stock options like in Johnson and Tian (2000) and time-varying parameters like volatiliy or the market risk β.

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